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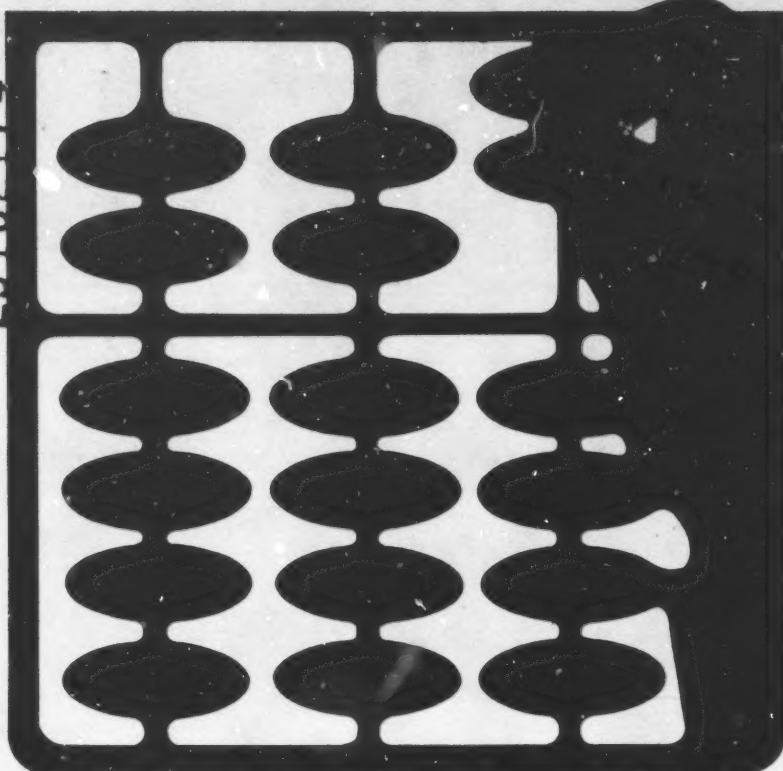
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ABSTRACT

This is one of a series of reports summarizing the results of the second mathematics assessment conducted by the National Assessment of Educational Progress (NAEP). Nine, 13- and 17-year-olds were assessed during the 1977-78 school year. The assessment measured achievement in various content areas at four levels of cognitive processes: knowledge, skill, understanding, and application. This report describes performance on the understanding items. Included are results for understanding of numbers and numeration, variables and relationships, geometry, measurement, and various other topics. The report also contains group results, age-level comparisons and observations on the results. The observations, consisting of a historical perspective, implications of the results, and recommendations, were made by a panel of people active in the field of mathematics education. Among the panel's five recommendations were: (1) An expanded definition of what is "basic" in mathematics is crucial to foster students' ability to cope with different types of mathematical tasks; (2) There is a need to promote understanding of concepts and to link skill development to problem-solving activities; and (3) There is a need for more consistent and comprehensive teacher education in the mathematics area. (MK)

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MATHEMATICAL UNDERSTANDING



SE 029 967

MATHEMATICAL UNDERSTANDING

SELECTED RESULTS FROM THE SECOND ASSESSMENT OF MATHEMATICS

Report No. 09-MA-04

by the
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FOREWORD

When the U.S. Office of Education was chartered in 1867, one charge to its commissioners was to determine the nation's progress in education. The National Assessment of Educational Progress (NAEP) was initiated a century later to address, in a systematic way, that charge.

Each year since 1969, National Assessment has gathered information about levels of education achievement across the country and reported its findings to the nation. NAEP surveys the education attainments of 9-year-olds, 13-year-olds and 17-year-olds in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure possible changes in education achievement. National Assessment has interviewed and tested more than 810,000 young Americans since 1969.

Learning-area assessments evolve from a consensus process. Each assessment is the product of several years of work by a great many educators, scholars and lay persons from all over the nation. Initially, these people design objectives for each subject area, proposing general goals they feel Americans

should be achieving in the course of their education. After careful reviews, these objectives are given to exercise (item) writers, whose task it is to create measurement instruments appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists, measurement experts and lay persons, they are administered to probability samples. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can make generalizations about the probable performance of all 9-year-olds in the nation.

After assessment data have been collected, scored and analyzed, National Assessment publishes reports to disseminate the results as widely as possible. Not all exercises are released for publication. Because NAEP will readminister some of the same exercises in the future to determine whether the performance level of Americans has increased, remained stable or decreased, it is essential that they not be released in order to preserve the integrity of the study.

ACKNOWLEDGMENTS

Many organizations and individuals have made substantial contributions to this report. Not the least of those to be gratefully acknowledged are the administrators, teachers and students who cooperated so generously during the collection of the data.

Special acknowledgment must go to Jane Armstrong of the National Assessment staff for coordination of development efforts and to Stuart Kahl and George Moreno for their assistance. Also to be acknowledged are the substantial contributions in guiding item development and report planning made by the members of the Mathematics Advisory Board: Sarah Herriot, Prince Jackson, Jeremy Kilpatrick, Donald Kreider, John LeBlanc and James Wilson.

Administration of the mathematics assessment was conducted by the Research Triangle Institute, Raleigh, North Carolina. Scoring

and processing were carried out by the Measurement Research Center, Iowa City, Iowa, and by the National Assessment staff.

Iris Carl, Thomas Carpenter, Robert Dilworth, Edgar Edwards Jr. and James Wilson contributed their time and knowledge interpreting the implications of the data. Their efforts helped greatly in putting the data into perspective.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must go to the following people: Ingrid Larsson and Charlotte Overbury for data processing support; Ava Powell and Marci Reser for production; Barbara Ward for editorial supervision. Statistical analyses for this report were supervised by Donald Phillips. The report was written by Stuart Kahl.



Roy H. Forbes
Director

INTRODUCTION

During the 1977-78 school year, the National Assessment of Educational Progress (NAEP) conducted its second mathematics assessment of the nation's 9-, 13- and 17-year-olds.¹ That assessment measured achievement in various content areas at four levels of cognitive processes — knowledge, skill, understanding and application. This report describes performance on the understanding items.

The 1977-78 Mathematics Assessment

A two-dimensional matrix was used to guide the development of items for the 1977-78 assessment (*Mathematics Objectives, Second Assessment, 1978, p. 17*). Cognitive process levels form one dimension of

the matrix, and mathematical content areas, the other, as shown below.

Items were developed to assess students' performance with respect to each of the "cells" of this matrix. Over 700 items were used.

Exercises in the knowledge category require the recall and recognition of mathematical ideas (facts, definitions, etc.). Skill exercises call for the routine manipulation of mathematical ideas by using previously learned and practiced algorithms in familiar situations. Skill items measure proficiency in carrying out algorithms, but do not require understanding of how or why they work. To demonstrate understanding of mathematical concepts, students must either explain or interpret mathematical ideas; thus, understanding items generally require some form of mental translation of knowledge. Applications items require the application of a sequence of cognitive processes — including knowledge,

¹The first NAEP assessment of mathematics was completed in 1972-73; changes in achievement between 1973 and 1978 are described in *Changes in Mathematical Achievement, 1973-78* (1979).

		CONTENT				
		Numbers and Numeration	Variables and Relationships	Geometry (Shape, Size and Position)	Measurement	Other Topics
PROCESS	Mathematical knowledge					
	Mathematical skill					
	Mathematical understanding					
	Mathematical application					

skills and understanding — to solve problems. Students must use reasoning and judgment, based on an understanding of the problem situation, to select the appropriate procedures.

This report focuses on mathematical understanding. Assessment exercises discussed in this report measure understanding of base-10 numeration, fractional numbers of different kinds, operations with numbers, relationships between variables, properties of geometric figures, measurement, probability, statistics, and graphs and tables.

Measuring Achievement

The mathematics assessment was administered to a national probability sample of 9-year-olds, 13-year-olds and 17-year-olds who were still attending school at the time of the assessment. The 9-year-olds were assessed in January and February of 1978; the 13-year-olds, in October through December of 1977; and the 17-year-olds, in March and April of 1978. About three-fourths of the 9-year-olds in the National Assessment sample were enrolled in the 4th grade, and about one-fourth were in the 3rd grade. Around three-fourths of the 13-year-olds were in the 8th grade, and most of the remainder were in the 7th grade. Approximately 75% of the 17-year-olds were in the 11th grade; about 15% were in the 10th grade and about 10% were in the 12th.

National Assessment reports scores for groups of students, not individuals. It also provides estimated percentages of correct responses for single items. When a report indicates that "85% of the 17-year-olds gave a correct response," it means that an estimated 85% of the 17-year-olds would have given a correct response if all the 17-year-olds in schools across the country had been assessed. Therefore, it was not necessary for each student to respond to every item. Each respondent completed only one exercise booklet of about 45 minutes in length. In 1977-78,

there were 7 booklets for 9-year-olds, 11 for 13-year-olds and 12 for 17-year-olds. Between 2,100 and 2,500 students responded to each booklet; approximately 17,000 9-year-olds, 27,000 13-year-olds and 27,000 17-year-olds participated in the 1977-78 mathematics assessment.

About half of the exercises in the second mathematics assessment are multiple-choice and about half are open-ended, meaning that students had to supply an answer rather than choose a response from a list of alternatives.

Responses to open-ended items were tabulated into various categories. These categories were defined based on results of exercise pretests conducted prior to the actual assessment. Scoring categories included likely errors and thus are useful in identifying frequently made mistakes. To minimize guessing, students were encouraged to write "I don't know" on the answer line on open-ended items or to select the "I don't know" option on multiple-choice items if they felt they did not know the answer to a question. In an effort to prevent reading ability from affecting performance, exercises were recorded on tape and read aloud to respondents as well as printed in the item booklets.

Additional Information on the Mathematics Assessment

The following reports, available from National Assessment, describe other aspects of the NAEP mathematics assessment: *Mathematical Knowledge and Skills* (1979) reports 1977-78 results on items assessing knowledge and skills; *Mathematical Applications* (1979) describes students' performance in 1977-78 on application exercises; *Changes in Mathematical Achievement, 1973-78* (1979) discusses changes in performance between the first and second assessments of mathematics. *Mathematics Objectives, Second Assessment* (1978) describes the objectives and the process-by-content matrix used to structure the second assessment.

Technical documentation of the 1977-78 assessment is presented in a released exercise set showing items released for publication, with national and selected group results; a microfiche data appendix providing additional

group results for released exercises; and a yearbook describing assessment procedures. Reports and data volumes describing results from the 1972-73 mathematics assessment are also available.

CHAPTER 1

UNDERSTANDING OF MATHEMATICAL CONCEPTS

The definition of mathematical understanding used by the National Assessment of Educational Progress (NAEP) states that understanding is demonstrated by performance on tasks requiring translation of knowledge — either by giving an illustration or explanation of a particular piece of knowledge or by translating information from one form to another. For example, a student might demonstrate understanding by explaining that right angles are an essential property of rectangles, by giving an equation to represent the information in a word problem or by providing a numerical equivalent of a fractional quantity expressed pictorially.

It is often difficult to design items that assess solely understanding, since in many cases the same mathematics exercise can be solved in a variety of ways. For instance, a student might complete a multiplication exercise by mechanically applying the multiplication algorithm with no understanding of why that algorithm works. Another student might purposefully use the distributive property of multiplication over addition (e.g., for 43×7 , $40 \times 7 = 280$, $3 \times 7 = 21$; thus the answer is 301), indicating an understanding of the process of multiplication. Of course, if a student has been drilled in the use of the distributive property, then its application may be reduced to the level of a practiced skill.

Since different solutions to an exercise might represent the use of different cognitive process levels by different students, it was, in some cases, difficult to determine the cognitive process level of an item. Because performance on some of the knowledge- and skill-level items reflects understanding or helps to put students' understanding into per-

spective, results for some of these items are included in this report. For a more complete discussion of students' mathematical knowledge and skills, see *Mathematical Knowledge and Skills* (1&79).

Numbers and Numeration

Whole Numbers

On knowledge exercises dealing with whole number place value, most 9-year-olds (83–90%) correctly wrote the standard numerals corresponding to the expanded notations for three-digit whole numbers (e.g., $165 = 1 \text{ hundred} + 6 \text{ tens} + 5 \text{ ones}$). Eighty to 87% of the 9-year-olds correctly identified what particular digits “stand for” (ones, tens, hundreds) in given numerals, and approximately half of them identified the digits found in particular “places” of given whole numbers.

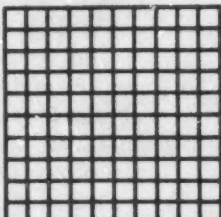
The exercise assessing understanding of place value, displayed in Exhibit 1, was answered correctly by 70% of the 9-year-olds. With a few exceptions, 9-year-olds' performance was generally higher on the knowledge and skill exercises dealing with place value and expanded notation than on this exercise, which utilizes a symbolic representation of expanded notation.

Thirteen-year-olds' performance on knowledge and skill exercises about place value was consistently high (82–96%). However, fewer at this age (73%) understood that 0 is the largest digit that could be in the $\square$ in $2\square 79$ if the number is to remain less than 2,100.

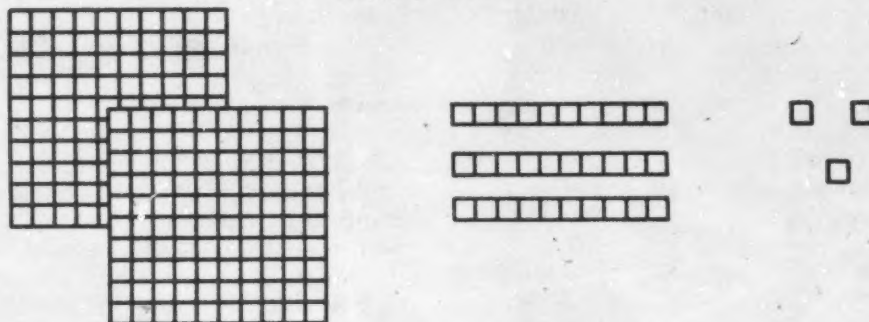
**EXHIBIT 1. Percentages of Response on Understanding Numeration
Exercise -- Age 9**

This is a postage stamp: ☐

This is a strip of 10 stamps: 

And this is a sheet of 100 stamps: 

Here is Teddy's stamp collection. How many stamps does Teddy have?



**Percent
Responding**

☐	110	5
☐	133	6
☐	232	4
☒	233	70
☐	332	7
☐	I don't know.	6†

† Figures do not total 100% because of rounding and/or nonresponse.

Fractions

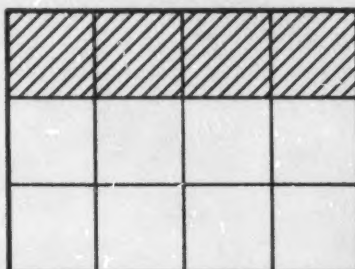
Exhibit 2 shows the results on two exercises dealing with understanding of fractional parts. Performance on these exercises improved substantially between ages 9 and 13. Many 9-year-olds may have had little formal experience with the concept of fractions. In the first exercise in the exhibit, many 9-year-olds simply indicated that the "top part" of

the box is shaded. Many 9-year-olds gave the exact number of shaded boxes or blue marbles rather than naming a fractional part. Notice that 58% of the 9-year-olds made this error on the second exercise shown in the exhibit.

Nine-year-olds' lack of experience with fractions is further illustrated in two unreleased exercises. Both exercises required the

EXHIBIT 2. Percentages of Response on Exercises Concerning Fractional Parts — Ages 9 and 13

What fractional part of this figure is shaded?



	Percent Responding	
	Age 9	Age 13
1/3 or 4/12*	20	82
4 or "top 4"	11	5
"top" or "top part"	24	..**

Shown a set of white and blue marbles, 9- and 13-year-olds were asked:

What fractional part of the marbles is blue?
(unreleased exercise)

	Percent Responding	
	Age 9	Age 13
Correct response	12	72
Exact number of blue marbles	58	8

*Correct response.

**This answer was not listed in the scoring guides for 13-year-olds.

students to shade a particular fraction (a simple common fraction) of a given figure; however, in one exercise the figure was divided into the same number of equal parts as the denominator of the fraction, and in the other, the number of equal parts in which the figure was divided is twice the number of the denominator. Sixty-two percent of the 9-year-olds shaded the correct fractional part when the number of parts equaled the number of the denominator, but only 9% of them shaded the correct number of parts when the number of parts in which the figure was divided is

twice the number of the denominator. In this case, 46% of them shaded a number of parts equal to the numerator of the fraction given. In other words, if 9-year-olds were asked to shade $\frac{3}{5}$ of a rectangle divided into 10 equal regions, many students at age 9 shaded $\frac{3}{10}$ rather than $\frac{3}{5}$ or $\frac{6}{10}$ of the figure.

Understanding of fractional parts increased for 13- and 17-year-olds. Seventy-six percent of the 13-year-olds and 88% of the 17-year-olds correctly identified the fraction that describes the total area shaded in a set of shaded and unshaded figures. Fifty-eight percent at age 13 and 82% at age 17 correctly selected the set of blue and white marbles depicting a certain fractional part.

Decimals

Several knowledge-level items about naming decimal numerals required students to identify the verbal forms of decimal fractions when given the numerals, or vice versa. Results for these exercises showed large differences between age groups. The percentages of correct responses were generally between 20 and 25% for 9-year-olds, between 65 and 80% for 13-year-olds and above 75% for 17-year-olds.

An item assessing understanding of decimals presented pairs of numbers, the numbers of each pair differing by $\frac{1}{2}$ (e.g., 2 and $2\frac{1}{2}$), and asked students to determine between which pair a given numeral with two decimal places belongs. Twenty-one percent of the 9-year-olds and 61% of the 13-year-olds selected the correct pair of numbers. This difference between 9- and 13-year-olds is consistent with that seen for knowledge-level exercises.

In other exercises concerning decimals, students were told that a certain number of objects out of a total number (either 10, 100 or 1,000) had a particular characteristic. Students were then asked to write the decimal representing the number of objects having that characteristic. Between 41 and 57% at

age 13 and between 49 and 69% at age 17 responded correctly to these questions. As

EXHIBIT 3. Percentages of Responses on Exercises About Computing With Decimals – Ages 13 and 17

Liz wrote this addition problem on her paper:

$$\begin{array}{r} 632 \\ 4.3 \\ .115 \\ \hline .788 \end{array}$$

Her teacher marked it wrong. Which one of the following best explains why it was marked wrong?

	Percent Responding	
	Age 13	Age 17
☐ Liz does not know her basic addition facts.	2	1
☒ Liz does not understand decimal places.	85	95
☐ Liz does not know the rule for "carrying" in addition.	2	1
☐ The teacher made a mistake; Liz's answer is correct.	10	3
☐ I don't know.	1	0

Fred wrote this subtraction exercise on his paper:

$$19.1 - .09 = 19.01$$

His teacher marked it wrong. Which one of the following best explains why it was marked wrong?

	Percent Responding	
	Age 13	Age 17
☐ Fred does not know his basic subtraction facts.	11	7
☐ Fred does not understand decimal places.	41	30
☐ Fred does not know the rule for "borrowing" in subtraction.	17	15
☒ The teacher made a mistake; Fred's answer is correct.	26	45
☐ I don't know.	5	3

might be expected, performance was slightly lower on items requiring zeros to the immediate right of the decimal point.

Exhibit 3 displays two exercises assessing students' understanding of decimals in a computational situation. Most students responded correctly to the first exercise shown; considerably fewer answered the second question correctly. The mistake in the first exercise is immediately apparent. However, in the second, students would have to align the numbers vertically and perform the computation to determine the correct answer.

Percent

Percentage has traditionally been a difficult concept for students at any cognitive level to deal with. Results on the percentage exercises in the mathematics assessment suggest that this is still the case. Exhibit 4 depicts the results on skill-level exercises involving percent. The first item – 30 is what percent of 60? – should be simple if one has an understanding of percent; results indicate that many students lack such understanding.

Exhibit 5 shows results for three other percentage exercises. Many students apparently understood what 25% means since 66% of the 13-year-olds and 83% of the 17-year-olds correctly shaded 25% of the squares shown (part C). However, considerably fewer teenagers could name the same percentage when given the shaded representation (part A). On both parts A and B in Exhibit 5, many students

EXHIBIT 4. Percentages of Correct Response on Selected Skills Exercises on Percent – Ages 13 and 17

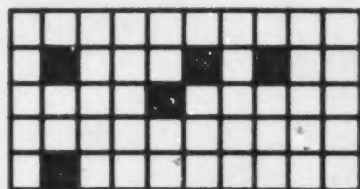
	Percent Correct	
	Age 13	Age 17
30 is what percent of 60?	35	58
What is 4% of 75?	8	27
12 is 15% of what number?	4	12
What is 125% of 40?	12	31
6 is what percent of 120?	6	16

EXHIBIT 5. Percentages of Response on
Exercise About Understanding Percent —
Ages 13 and 17



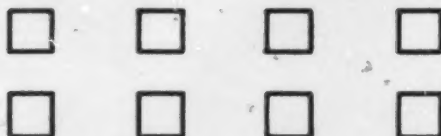
A. What percent of the circles is shaded?

	Percent Responding	
	Age 13	Age 17
25%*	28	52
1/4, 2/8 or .25	38	29



B. What percent of the rectangle is shaded?

	Percent Responding	
	Age 13	Age 17
10%*	18	38
1/10 or 5/50 or .1	31	23



C. Shade 25% of the squares.

	Percent Responding	
	Age 13	Age 17
2 squares shaded*	66	83
4 squares shaded	9	4

*Correct response.

gave the correct common fraction or decimal represented by the figure but not the percentage that was asked for.

Another exercise, administered only to 13-year-olds, asked what percentage of the U.S. population would be 20 years of age or older if 37% of the population were under 20 years of age. Sixty-nine percent of the 13-year-olds correctly selected the response that "63% of the U.S. population would be 20 years of age or older."

Operations

Modeling. One way to help students understand mathematical operations is through mathematical models, or visual representations of the operations. Exhibit 6 shows results on exercises in which sets of objects were used to display the various operations. Fairly high percentages recognized that a representation showing two sets of objects being joined stands for addition. Nine-year-olds were only slightly less successful in writing the proper multiplication sentence for a set of objects arranged in rows. Division proved more difficult — 26% at age 9 and 52% at age 13 wrote the proper division sentence for a set of objects divided into equal groups.

The number line is often used as a visual representation of mathematical operations. One must be cautious in using results for exercises concerning the number line as an indication of the understanding of operations, since performance on these exercises is dependent on students' familiarity with the conventions of the number line.

Many students at ages 9 and 13 probably have not used a number line before. At best, only one-third of the 9-year-olds and under one-half of the 13-year-olds correctly answered questions involving number lines (Exhibit 7). On the first exercise shown in the exhibit, one-fourth of the 9-year-olds and almost half of the 13-year-olds correctly wrote the addition sentence that corresponds to the number line model shown. However, in Exhibit 6, 74% and 90% of the 9-year-olds and 13-year-olds, respectively, recognized that a model showing sets of objects being joined stands for addition. The numbers used in these problems were the same.

EXHIBIT 6. Percentages of Correct Response on Exercises Involving Models of Operations — Ages 9 and 13

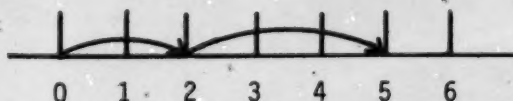
Given	Required	Percent Correct	
		Age 9	Age 13
Model of addition showing objects being combined and choice of number sentences involving different operations	Select number sentence corresponding to model	74	90
Model of multiplication consisting of rectangular array of objects	Write corresponding multiplication number sentence	67	**
Model of division showing a set of objects divided into equal subsets	Write corresponding division number sentence	26	52

***This exercise was not administered at age 13.*

EXHIBIT 7. Percentages of Correct Response on Exercises Involving Number Lines — Ages 9 and 13

Given	Required	Percent Correct	
		Age 9	Age 13
Model of addition using number line	Write corresponding addition sentence	25	48
Addition sentence and incomplete model using number line	Complete model (i.e., draw second arrow)	32	49
Model of subtraction using number line	Write corresponding subtraction sentence	14	33
Multiplication sentence and choice of models using number line	Selection of model corresponding to given multiplication sentence	33	48
Model of division using number line	Write corresponding division sentence	22	24

The most common errors indicate students' lack of familiarity with number lines. Often, students interpreted the numbers pointed to by the arrows as the numbers making up the problem, not realizing that the number the last arrow points to is the *result* of the computation. For example, the number line model for $2 + 3 = 5$ looks like this:



When asked to write the number sentence corresponding to this number line, students making such an error wrote $2 + 5 = 7$. Forty-five

percent of the 9-year-olds and 39% of the 13-year-olds made this type of error on the first exercise shown in Exhibit 7. Students had more difficulty with subtraction and division models than with addition and multiplication models.

Properties of operations. Understanding the properties of operations can help students apply computational skills. For example, the computation $(37 \times 5) \times 2 = ?$ can be answered fairly easily by using the associative property: $37 \times (5 \times 2) = ?$ Approximately one-third of the 13-year-olds and one-half of the 17-year-olds completed this computation correctly in the short time allowed. Many students either might not know the properties of operations or might not understand when their application would be helpful.

Several items assessed understanding of the properties of operations. Seventy percent of the 9-year-olds correctly answered the following item on the commutative property of multiplication: "a and b stand for two whole numbers. If $a \times b = 84$, then $b \times a = ?$ " Eleven percent of the 9-year-olds incorrectly answered "48."

Sixty-four percent of the 13-year-olds correctly selected the algebraic expression corresponding to a verbal explanation of the commutative property of addition. Thirty-six percent of the 13-year-olds and 52% of the 17-year-olds selected the algebraic expression corresponding to a verbal description of the associative property of addition. Forty-eight percent at age 13 and 64% at age 17, when shown the associative principle expressed symbolically, knew that it applies to addition and multiplication, but not to subtraction or division.

Variables and Relationships

Although variables and relationships are most often thought of in connection with algebra, they are introduced in the mathematics curriculum before formal instruction in algebra. Most 13-year-olds would have some ex-

perience in substituting particular values for variables in algebraic expressions and in solving simple linear equations and inequalities.

The knowledge and skills report indicates considerable growth between ages 9 and 13 in ability to deal with simple linear equations and inequalities. For example, 15% of the 9-year-olds and 54% of the 13-year-olds named the number that would go in the box to make the number sentence $\square - 19 = 32$ true. Twenty-six percent of the 9-year-olds and 66% of the 13-year-olds selected the number, from a list of alternatives, that would make $5 + \square < 12$ true.

Approximately 72% of the 17-year-olds have taken at least one-half year of Algebra I, and approximately 37% of them have taken at least one-half year of Algebra II. The algebraic skills exercises administered to 17-year-olds dealt with skills typically taught in either Algebra I or Algebra II. As noted in the knowledge and skills report, between 50 and 65% of the 17-year-olds could perform simple algebraic manipulations such as combining like terms, and about one-third of them solved simple linear equations like $30 = 2/5c + 10$ and $3x + 6 - 14 = x + 2$. Less than one-fourth of the 17-year-olds succeeded at more complex manipulations such as factoring quadratic expressions.

Relationships Between Variables

In addition to being able to manipulate variables, students should be able to understand the relationships between variables. Exhibit 8 depicts the results on two exercises concerning variables. Understanding that the relationship between a and b cannot be determined increased substantially between ages 9 and 13 and 13 and 17. Teenagers had more difficulty with the second example — about one-third of the 13-year-olds and about one-half of the 17-year-olds answered correctly.

Fifty-seven percent of the 13-year-olds displayed understanding of the transitive property (for example, if $a = b$ and $b = c$, then a

EXHIBIT 8. Percentages of Response on Exercises Requiring Understanding of Variables and Relationships – Ages 9, 13 and 17

If $a > 5$ and $b > 5$, then

	Percent Responding		
	Age 9	Age 13	Age 17
☐ a equals b.	30	32	16
☐ a is greater than b.	10	4	2
☐ b is greater than a.	14	2	2
☒ There is not enough information to determine the relation between a and b.	27	57	76
☐ I don't know.	16†	5	3†

$$(n + 5) - 5 = n$$

What values of n make this sentence TRUE?

	Percent Responding	
	Age 13	Age 17
☐ 0 only	29	25
☐ 0 and 5 only	20	16
☒ Any number	34	48
☐ No number	5	6
☐ I don't know.	12	5

†Figures do not total 100% because of rounding and/or nonresponse.

$= c$; if $a > b$ and $b > c$, then $a > c$) – correctly answering that if x is less than a given value and y is less than x , then y must be less than the given value. However, on an exercise administered to all three age groups, 50% of the 9-year-olds, 72% of the 13-year-olds and 81% of the 17-year-olds showed an understanding of transitivity with respect to the relation “is older than.” It may be easier for students to deal with real objects than with abstract variables such as x and y .

Exhibit 9 shows the results on an exercise requiring students first to determine the rela-

tionship between members of ordered pairs in a table, and then to complete the table. Success at finding y was considerably higher when x equals 3 than when x equals the variable n .

Results for 13- and 17-year-olds were remarkably similar on the exercise displayed in Exhibit 10. While students might gain experience in manipulating equations between the ages of 13 and 17, ability to understand the meaning of this equation did not improve between the two ages.

EXHIBIT 9. Percentages of Response on Exercise on Completing Table of Ordered Pairs – Ages 13 and 17

x	y
1	6
3	
4	9
7	12
n	

A. What is the value of y in the above table when $x = 3$?

Percent Correct	
Age 13	Age 17
35	58

B. What is the value of y in the above table when $x = n$?

	Percent Responding	
	Age 13	Age 17
$n + 5^*$	2	31
15	11	8
0	10	4
5	7	3
Other unacceptable	27	22
I don't know.	25	20

*Correct response.

**EXHIBIT 10. Percentages of Response on
Exercise Involving a Linear Equation —
Ages 13 and 17**

Some people suggest the following formula be used to determine the average weight for boys between the ages of 1 and 7:

$$W = 17 + 5A$$

where W is the average weight in pounds and A is the boy's age in years.

According to this formula, for each year older a boy gets, how much more should he weigh?

	Percent Responding	
	Age 13	Age 17
☒ 5 pounds	54	54
☐ 17 pounds	21	22
☐ 22 pounds	17	14
☐ I don't know.	8	9†

†Figures do not total 100% because of rounding and/or nonresponse.

Several exercises dealt with direct and inverse variation. Fifty-eight percent of the 17-year-olds knew that if $y = 15 - x$, then as x increases, y decreases; and 47% of the 17-year-olds realized that for an equation of the form $y = a/x$, as x increases, y decreases. Given a quadratic function of the form $y = ax^2$, about one-fifth of the 17-year-olds knew that if x is doubled, y would be multiplied by 4. Many 17-year-olds (45%) said that y would be doubled. Similarly, 17% of the 17-year-olds knew that if the radius of a circle is doubled, the area becomes four times as great; most students (59%) said that the area would be doubled.

**Translation From Verbal to
Symbolic Form**

To solve mathematical problems, students must understand the situation presented in a problem. To assess this understanding, assessment items asked students to translate information presented verbally into algebraic expressions or equations.

Several exercises asked students to identify the expression depicting the situation given in the problem. Sixty-four percent of the 13-year-olds and 82% of the 17-year-olds correctly gave the algebraic expression for the length of a piece of board remaining after a certain length was cut off the original board. Seventy-eight percent of the 13-year-olds correctly selected " $x > y$ " as the expression describing the following situation: "Tom has x marbles and Mary has y marbles. Mary has more marbles than Tom."

Exhibit 11 reports the performance of the three age groups on exercises requiring them to translate the information presented verbally into number sentences. Notice that 18% of the 9-year-olds mistakenly chose the second alternative in part B — adding the numbers given in the problem.

A similar problem involved a classic division situation. Each of the four response op-

**EXHIBIT 11. Percentages of Response on
Exercise Involving Translation (Verbal to
Symbolic) — Ages 9, 13 and 17**

	Percent Responding		
	Age 9	Age 13	Age 17
A. Paul has 21 stamps in his collection. He buys 54 more. Which number sentence tells how many he has after he buys them?			
☐ $54 - 21 = \square$	9	2	1
☒ $21 + 54 = \square$	66	92	96
☐ $2(21 + 54) = \square$	6	2	1
☐ $54 \times 21 = \square$	6	2	0
☐ I don't know.	10†	1†	1†
B. Mike took 15 cents to the store. He saw a pen that costs 23 cents. Which number sentence tells you how much more he needs to buy the pen?			
☐ $23 + \square = 15$	9	4	2
☐ $15 + 23 = \square$	18	5	3
☒ $15 + \square = 23$	50	84	90
☐ $\square + 23 = 15$	6	3	2
☐ I don't know.	13†	3†	2†

†Figures do not total 100% because of rounding and/or nonresponse.

tions contained the same numbers, but with a different operational sign. Forty-four percent of the 9-year-olds and 82% of the 13-year-olds correctly selected the division sentence as corresponding to the verbal information.

Thirteen- and 17-year-olds were asked to answer this exercise: "Carol earned D dollars during the week. She spent C dollars for clothes and F dollars for food. Write an expression using D, C and F that shows the number of dollars she had left." Seventeen percent at age 13 and 45% at age 17 wrote an expression accurately describing the situation.

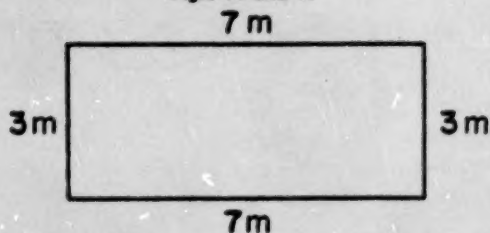
Geometry

Properties of Figures

Results on knowledge-level exercises indicate that students were reasonably proficient at identifying plane and solid figures. However, generally fewer students displayed understanding of the properties that define figures. For example, 93% of the 9-year-olds correctly identified a square from a set of plane figures. But, when shown a figure identified as a square and given the length of one side, only two-thirds of the 9-year-olds realized that an adjacent side must be the same length.

The exercise shown in Exhibit 12 measured students' understanding of the properties that generally distinguish rectangles from parallelograms. Many 13- and 17-year-olds seemed to think that equal, opposite sides or parallel, opposite sides guarantee that a figure is a rectangle; relatively few (14% at age 13 and 20% at age 17) realized that a rectangle must also contain right angles. Another exercise regarding rectangles asked students to determine whether a quadrilateral with opposite sides of equal length is a rectangle. Eleven percent at age 13 and 19% at age 17 selected the correct answer, which is to determine whether the diagonals of the figure are equal. However, again, many teenagers (53% of the 13-year-olds and 48% of the 17-year-olds) incorrectly selected the option that stated nothing needs

EXHIBIT 12. Percentages of Response on Exercise Involving Properties of Rectangles – Ages 13 and 17



In the figure above, the lengths of the sides are shown. Which of the following guarantees the figure is a rectangle?

	Percent Responding	
	Age 13	Age 17
☐ The opposite sides are congruent.	26	31
☐ The opposite angles are congruent.	9	8
☒ The angles are right angles.	14	20
☐ The opposite sides are parallel.	40	34
☐ I don't know.	10†	6†

† Figures do not total 100% because of rounding and/or nonresponse.

to be done because a figure with equal, opposite sides is a rectangle.

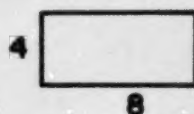
Several exercises investigated understanding of the properties of triangles. On a knowledge exercise, 54% of the 17-year-olds knew that the sum of the angles of a triangle is 180° . When asked to demonstrate their understanding of this property by determining which of several sets of numbers could *not* be measures of the angles of a triangle, 20% of the 13-year-olds and 42% of the 17-year-olds correctly selected the one set that did not total 180° . Other exercises concerned the lengths of the line segments needed to form a triangle. When shown several sets of line segments, 54% of the 9-year-olds and 64% of the 13-year-olds correctly selected the set that could not form a triangle (the sum of the lengths of two line segments was less than the length of the third). However, when given several sets of

three numbers and asked which set could not be the lengths of a triangle, only 11% of the 13-year-olds and 18% of the 17-year-olds answered correctly. Performance was much higher on the exercise in which students saw the line segments and could visualize whether or not the segments could form a triangle than on the exercise giving only the lengths of the line segments. On the latter exercise (giving only the lengths of the line segments), stu-

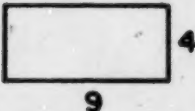
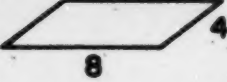
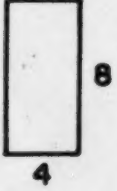
dents either had to recall the theorem that the sum of two sides of a triangle is greater than the third side and recognize that it applies in this situation or realize that they needed to visualize the line segments corresponding to the given lengths.

A series of items administered to 17-year-olds concerned understanding of the angle configurations of various types of triangles.

EXHIBIT 13. Percentages of Response on Congruence Exercise — Ages 13 and 17



Is each figure below congruent to the figure above?

		Percent Responding		
		Yes	No	I don't know.
A.		☐	☒	☐
	Age 13	31	65	3†
	Age 17	26	71	2†
B.		☐	☒	☐
	Age 13	39	58	3
	Age 17	22	76	2
C.		☐	☒	☐
	Age 13	63	32	3†
	Age 17	68	28	3†
D.		☒	☐	☐
	Age 13	89	8	3
	Age 17	93	4	2†
E.		☐	☒	☐
	Age 13	43	50	6†
	Age 17	42	50	6†

†Figures do not total 100% because of rounding and/or nonresponse.

Given a triangle defined as having a particular obtuse angle (the triangle was not shown), 60% knew that the triangle could not be a right triangle; 53% knew that it might be isosceles; 47% knew that it could not be equilateral; and 54% knew the sum of the remaining angles.

On an exercise concerning properties of a circle, and showing the circle with several line segments drawn in its interior, 51% and 72% of the 13- and 17-year-olds, respectively, identified a diameter as being twice the length of a radius.

Congruence and Similarity

The concepts of congruence and similarity are important in working with geometric figures. Given several pairs of plane figures, most students at ages 9 and 13 (82% and 94%, respectively) correctly identified the pair in which the figures had the "same size and shape." However, the results on the exercise shown in Exhibit 13 indicate that fewer students understand the term "congruence." Most students at ages 13 and 17 recognized that the figure shown in part D is congruent to the given figure. However, many students apparently assumed that figures with the same numbers for dimensions are congruent even though the figures are different. Sixty-three percent at age 13 and 68% at age 17 indicated that the figure shown in part C is congruent

to the given figure; slightly over 40% at each age thought that the figure in part E is congruent to the given figure.

Eighty-six percent of the 9-year-olds and 94% of the 13-year-olds selected the figure "similar to" a given triangle from a set of plane figures including three different triangles and a rectangle. Thirteen- and 17-year-olds did quite well in determining whether or not various pairs of plane figures were similar (Exhibit 14).

Measurement

Results on knowledge-level items indicate that many students are familiar with metric units, the appropriateness of different units for different tasks, and equivalent units. For example, 36% at age 9, 63% at age 13 and 69% at age 17 correctly identified the largest of a set of metric units of length. Eighty-eight percent of the 9-year-olds knew that miles are the most appropriate English unit for describing long distances. Many teenagers (87% at age 13 and 92% at age 17) knew how many inches there are in a foot, and 72% of the 17-year-olds selected the English unit closest in length to a given metric unit.

Exercises dealing with measurement skills required students to read various measurement instruments and to make linear measurements. As might be expected, performance was higher on tasks requiring gross measurements than on those demanding finer distinctions. Many students appear to lack understanding of the concept of precision of measurement. Sixty-five percent of the 17-year-olds knew that the actual length of an object that was measured to the nearest of a particular unit could be anywhere from one-half unit less to one-half unit more than the given measurement. One-fourth of the 17-year-olds, however, stated that the actual length could be anywhere from one unit less to one unit more than the given measurement.

While many students appear to have some understanding of what is meant by area, their understanding of the mathematical relation-

EXHIBIT 14. Percentages of Correct Response
on Exercises Concerning Similarities —
Ages 13 and 17

Students were asked whether the figures in each pair of figures were similar:

	Percent Correct	
	Age 13	Age 17
Squares of different sizes (yes)	90	96
Nonsimilar rectangles (no)	83	90
Nonsimilar triangles (no)	77	80
Circles of different sizes (yes)	88	93

ship between the dimensions of figures and their areas is not so well-developed. Twenty-eight percent of the 9-year-olds and 71% of the 13-year-olds found the area of a rectangle when dimensions were given and unit squares were drawn in, so students could count the squares rather than calculate the number of units. However, on a related exercise, only 4% at age 9, 51% at age 13 and 74% at age 17 found the area of the same rectangle when the unit squares were not drawn in.

Another exercise concerning area displayed a rectangle with dimensions and unit squares drawn in. Students were shown four other rectangles, each with different dimensions than the original rectangle and each with unit squares drawn in. Nineteen percent of the 9-year-olds and 62% of the 13-year-olds correctly selected the rectangle with the same area as the given rectangle.

On an exercise assessing understanding of the concept of volume, 66% of the 9-year-olds and 87% of the 13-year-olds realized that, given a small container and a larger one, more of the small than the larger containers would be needed to fill the same volume.

Other Topics

Probability

Consumer information, particularly information pertaining to health or safety, is often presented in terms of probability. A number of assessment exercises measured students' understanding in this area. Most of these exercises concerned classic probability situations, such as drawing objects from containers, flipping coins, rolling dice or spinning spinners.

One exercise measured whether students knew if their chances of picking a particular object from a set of objects are greater if the set contained a small or a large number of objects. Fifty-seven percent of the 9-year-olds, 84% of the 13-year-olds and 86% of the 17-year-olds correctly indicated that their chances of picking a particular object from the smallest set of objects shown would be greater.

The results on another probability exercise are shown in Exhibit 15. Many respondents at each age mistakenly selected the first option, apparently basing their decision on the fact

EXHIBIT 15. Percentages of Response on Probability Exercise
Involving Probabilities of Picking a Particular Object — Ages 9, 13 and 17

In a bag, there are 8 red marbles, 7 green ones, and 6 blue ones. Jane is going to reach into the bag and take out one marble without looking. Bill thinks Jane will get a red marble, and Jane thinks she will not. Who is more likely to be right?

	Percent Responding		
	Age 9	Age 13	Age 17
☐ Bill is more likely to be right.	40	46	37
☒ Jane is more likely to be right.	28	29	40
☐ Bill and Jane are equally likely to be right.	24	23	21
☐ I don't know.	7†	2	1†

† Figures do not total 100% because of rounding and/or nonresponse.

that there are more red marbles than either green or blue ones and failing to realize that they needed to consider the number of red marbles compared with the total number of other marbles.

On multiple-choice exercises about a number cube, 55% of the 13-year-olds and 75% of the 17-year-olds realized that the probability of rolling a particular number was $1/6$. Fewer teenagers — 40% of the 13-year-olds and 59% of the 17-year-olds — recognized that the probability of *not* rolling a particular number was $5/6$.

Other exercises dealt with the independence of events. As noted previously, 55% of the 13-year-olds and 75% of the 17-year-olds correctly selected the probability of rolling a particular number on a number cube. However, percentages dropped somewhat (to 44% at age 13 and 65% at age 17) when students were told that the same number had been rolled several times in a row and were asked to select from a list of options the probability of getting that number again on the next roll. When the second question (the probability of a number occurring after it had been rolled several times) was presented as an open-ended item, percentages of success were considerably lower — 17% at age 13 and 28% at age 17 responded correctly.

Exhibit 16 displays another item involving independence of events. Seventeen-year-olds seem to have understood the concept of independence of events, since nearly 70% either responded correctly or said that the chances of a boy or girl being born are equal. Many students may not have realized that the actual probabilities are *not* equal, but instead, slightly favored boys.

Two exercises asked students to indicate the number of times heads would be most likely to come up if a coin were flipped a certain number of times. On these items, 41% and 52% of the 9-year-olds, 66% and 72% of the 13-year-olds and 74% and 77% of the 17-year-olds selected the correct response.

EXHIBIT 16. Percentages of Response on Probability Exercise Involving Independence of Events — Age 17

In the United States, of every 1000 babies born, 515 are boys. In a certain U.S. hospital, the last 27 babies born have been girls.

The next baby born in that hospital will

	Percent Responding
☐ almost certainly be a girl (over 80% chance).	4
☐ almost certainly be a boy (over 80% chance).	14
☒ have a slightly greater chance of being a boy than a girl.	30
☐ have a slightly greater chance of being a girl than a boy.	8
☐ have an equal chance of being a girl or boy.	38
☐ I don't know.	5†

† Figures do not total 100% because of rounding and/or nonresponse.

Three exercises concerned a set of spinners. The spinners were divided into numbered sections of varying sizes. Students were told they would either win or lose if certain numbers were spun and were asked to indicate in each case the spinner they would choose to give them the best chance of winning. Students had to realize that the area of the spinner covered by sections labeled by the desired number is related to the probability of that number being spun. Percentages of correct responses ranged from 26% to 56% for 9-year-olds, from 67% to 79% for 13-year-olds and from 81% to 86% for 17-year-olds.

Exhibit 17 shows an exercise requiring students to match observed results with the spinner that would be most likely to produce those results. About two-fifths of the 13-year-olds and slightly over three-fifths of the 17-year-olds were successful at this task.

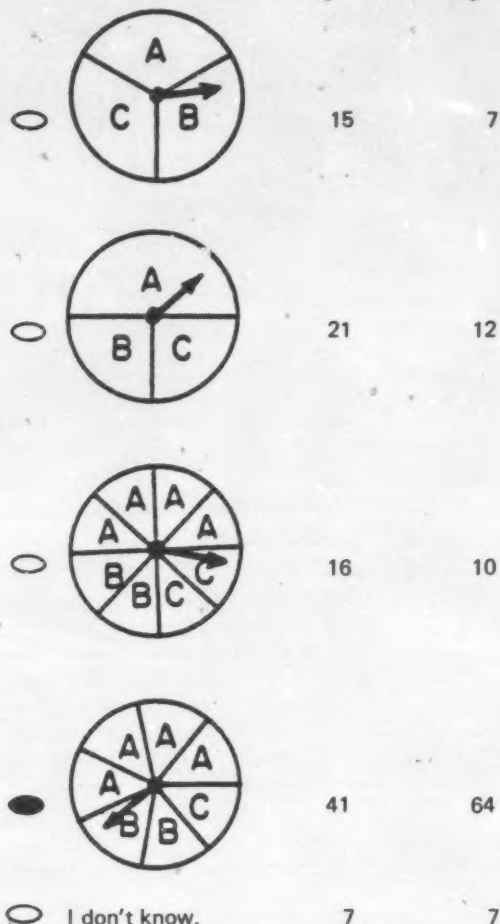
EXHIBIT 17. Percentages of Response on Probability Exercise Involving Experimental Data — Ages 13 and 17

Kim spun a spinner 100 times and made a record of her results.

OUTCOME	A	B	C
NUMBER OF TIMES	55	30	15

Which spinner is most likely the one that Kim used? Fill in the oval beside the one you choose.

Percent Responding
Age 13 Age 17



Statistics

Understanding of statistical terminology does not appear to increase very much between ages 13 and 17. Given the ages of five children in rank order, 42% of the 13-year-olds and 43% of the 17-year-olds identified the median age, 24% of the 13-year-olds and 26% of the 17-year-olds identified the mode and 6% of the 13-year-olds and 12% of the 17-year-olds identified the mean. Interestingly, 6% of the 9-year-olds, 51% of the 13-year-olds and 71% of the 17-year-olds correctly identified the "average" of the same five numbers. Thus, many students appear to be familiar with the term "average" but not the term "mean." When given a longer list of larger numbers not in rank order, only 10% of the 13-year-olds and 16% of the 17-year-olds correctly named the median.

Graphs and Tables

An informed citizen and consumer must be able to read and interpret information presented in various graphic and tabular forms. National Assessment exercises in this category involved circle, bar and line graphs as well as tables.

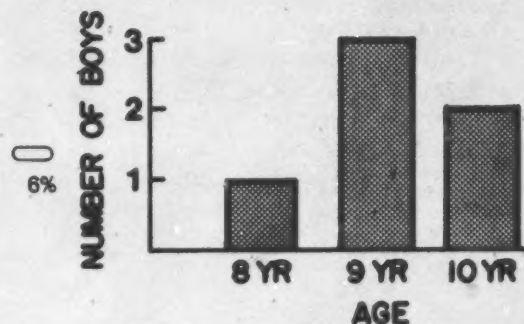
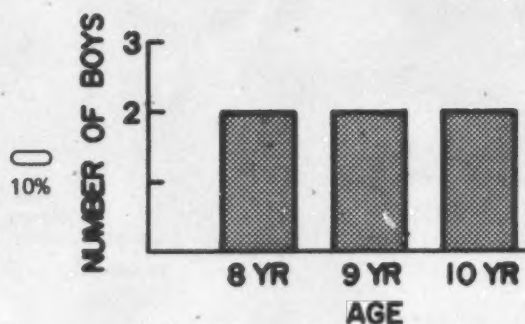
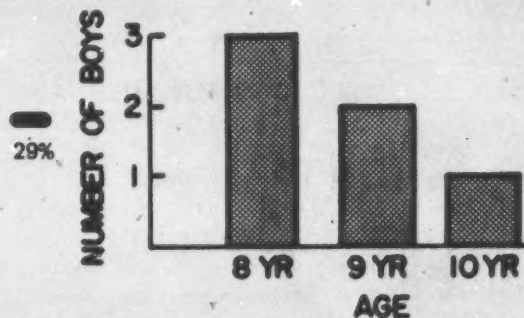
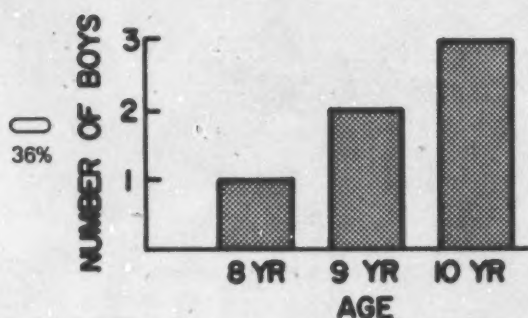
The results for knowledge- and skill-level exercises suggest that by age 13, students' abilities to read graphs and tables and to interpret the information are fairly well-developed. However, it is difficult to make generalizations about relative performance on exercises involving different kinds of graphs or tables and different cognitive levels, since so many factors can have an influence — the format of the graph or table, the number of variables presented and the visual/perceptual characteristics of the graph or table, to name a few.

In the exercise shown in Exhibit 18, 9-year-olds were asked to identify the graph that depicts the information given. More 9-year-olds selected the first option, in which the bars increase in length as the ages of the boys increase, than the correct option (36%

EXHIBIT 18. Percentages of Response on Bar Graph Exercise — Age 9

Here are the ages of six boys:

10, 9, 9, 8, 8, 8



16%† I don't know.

† Figures do not total 100% because of rounding and/or nonresponse.

compared with 29%). The first option would be an obvious choice for students who did not understand what the vertical axis represents.

A line graph exercise for 17-year-olds referred to a graph with rate of speed on one axis, elapsed time on the other axis and a separate line for each of three vehicles. Eighty-one percent of the 17-year-olds identified the vehicle that was traveling the fastest at the end of a particular period of time, and

70% of them correctly identified the vehicle that reached a particular speed in the shortest time. Another graph exercise for 17-year-olds showed dollars on one axis, years on the second axis and separate lines for a company's income and expenses. Forty-four percent of the 17-year-olds identified the year in which the company made the largest profit. Such interpretations require considerable understanding of what is shown by the graphs.

CHAPTER 2

GROUP RESULTS AND AGE-LEVEL COMPARISONS

To examine the relationships between the characteristics of students and mathematics achievement, National Assessment analyzed results for various groups within the national population. Some of these groups are defined by students' backgrounds or personal characteristics; others are related to their school experiences. In addition to discussions of various groups' results at each age level, this chapter makes comparisons between age groups based on performance on identical sets of mathematical understanding items administered to more than one age group.

Group Results

Groups related to students' background characteristics were based on the following factors: region of the country, sex, race, level of parents' education, and type and size of community lived in. Groups based on the respondents' school experience were defined by: grade level in school and, for 17-year-olds, types of mathematics courses taken. Definitions of groups reported upon appear in Appendix A.

Description of the Data

Group performance is described in terms of average performance on the set of understanding exercises and is compared with the national performance level on that item set. Exhibits 19-25 and 27 display differences between the various types of groups and the nation on the entire set of mathematical understanding items. In these exhibits, the zero line represents the national performance level. The solid, horizontal line within each box for each

group represents the difference between the group's mean and the national mean. A positive difference means that the group performed above the nation, and a negative difference, below it. The bar surrounding each mean represents two standard errors above and below the mean achievement for the group.

A standard error of the sample mean is an estimate of the sampling variability among the means of all possible samples; it is used to estimate the precision of the mean obtained in a particular sample. Intervals of two standard errors below to two standard errors above a particular statistic would include the average of the statistic in approximately 95% of all possible samples of the same size. An interval computed in this way is a 95% confidence interval. If the standard error bar for a particular group mean does not cross the national line, one can be reasonably sure that the group mean is different from the national mean.

It should be remembered that differences in achievement levels cannot be ascribed solely to membership in the particular group being discussed. Research has shown that any number of socioeconomic, school-related and environmental factors contribute to achievement, and caution must be taken not to attribute a cause-and-effect relationship between membership in a specific group and performance.

Background Variables

The following sections describe patterns of results on understanding exercises for the vari-

ous groups. National Assessment found that similar patterns of performance prevailed across age levels — the same groups were generally either above or below national levels, although in some cases tendencies became more marked for the older students. Appendix B presents average group results relative to the nation for each group for the entire set of understanding items.

Region. As seen in Exhibit 19, the relative achievement levels of the geographic regions were fairly consistent across ages. The Northeast (NE) and Central (C) regions were above the nation; the West (W) was generally below the nation, and the Southeast (SE) was usually below the West.

Sex. Patterns of differences between the mathematical performance of the sexes on understanding items were not as consistent as patterns for other variables. Performance differences between the sexes were very small or nonexistent at the 9- and 13-year-old levels. Differences were more pronounced at age 17, with males (M) outperforming females (F) by a little less than 4 percentage points (Exhibit 20).

Different course backgrounds of males and females have been cited as a possible cause of the differences in mathematical performance. Forthcoming National Assessment documents will report on the extent to which course enrollment patterns can account for performance differences. In recent years, researchers have referred to a 2:1 ratio of males to females enrolled in the more advanced mathematics courses. The 1977–78 assessment found a 3:2 ratio of 17-year-old males to 17-year-old females who had taken advanced mathematics courses beyond Algebra II. Also, the percentage of the total student population taking such courses is not very large. For example, 5% of the 17-year-old males had taken Calculus or Pre-calculus, compared with 3% of the females. Differences in enrollments between the sexes were not found for lower-level courses including Algebra I, Geometry and Algebra II. It may be that differences in the mathematics enroll-

ment patterns of the males and females are diminishing, and it is not likely that the differences that do exist can account for the total performance difference.

Race. The average achievement of whites (W) was higher than that of Hispanics (H) and blacks (B) at all three age levels. Hispanic achievement tended to be higher than that of blacks (Exhibit 21). The differences between the three racial groups reported upon increased with age. For example, the gaps between whites and blacks for mathematical understanding were approximately 13, 18 and 20 percentage points at ages 9, 13 and 17, respectively.

There were some important differences between blacks and whites with respect to mathematics course enrollment patterns, especially for lower-level academic mathematics courses. Three-fourths of 17-year-old whites had taken Algebra I, compared with 55% of 17-year-old blacks. The percentages of 17-year-old whites and blacks who had taken Geometry were 55% and 31%, respectively, and for those who had taken Algebra II, 38% and 24%, respectively.

Parental education. At all three age levels, students whose parents had more education performed better than those whose parents had less education (Exhibit 22). The performance differences between the parental-education groups increased with the age of the students. For example, on the understanding exercises, 9-year-olds whose parents had some education beyond high school (PH) performed approximately 13 percentage points higher than 9-year-olds whose parents did not graduate from high school (NG). The corresponding gap for 17-year-olds was approximately 18 percentage points. The performance of students whose parents graduated from high school but did not continue their education (GH) was between that of the other two groups.

Type of community. National Assessment reports upon three types of communities: advantaged urban (AU), disadvantaged urban

EXHIBIT 19. Differences in Mean Percentages Between Regional Groups
and the Nation on Understanding Items – Ages 9, 13 and 17

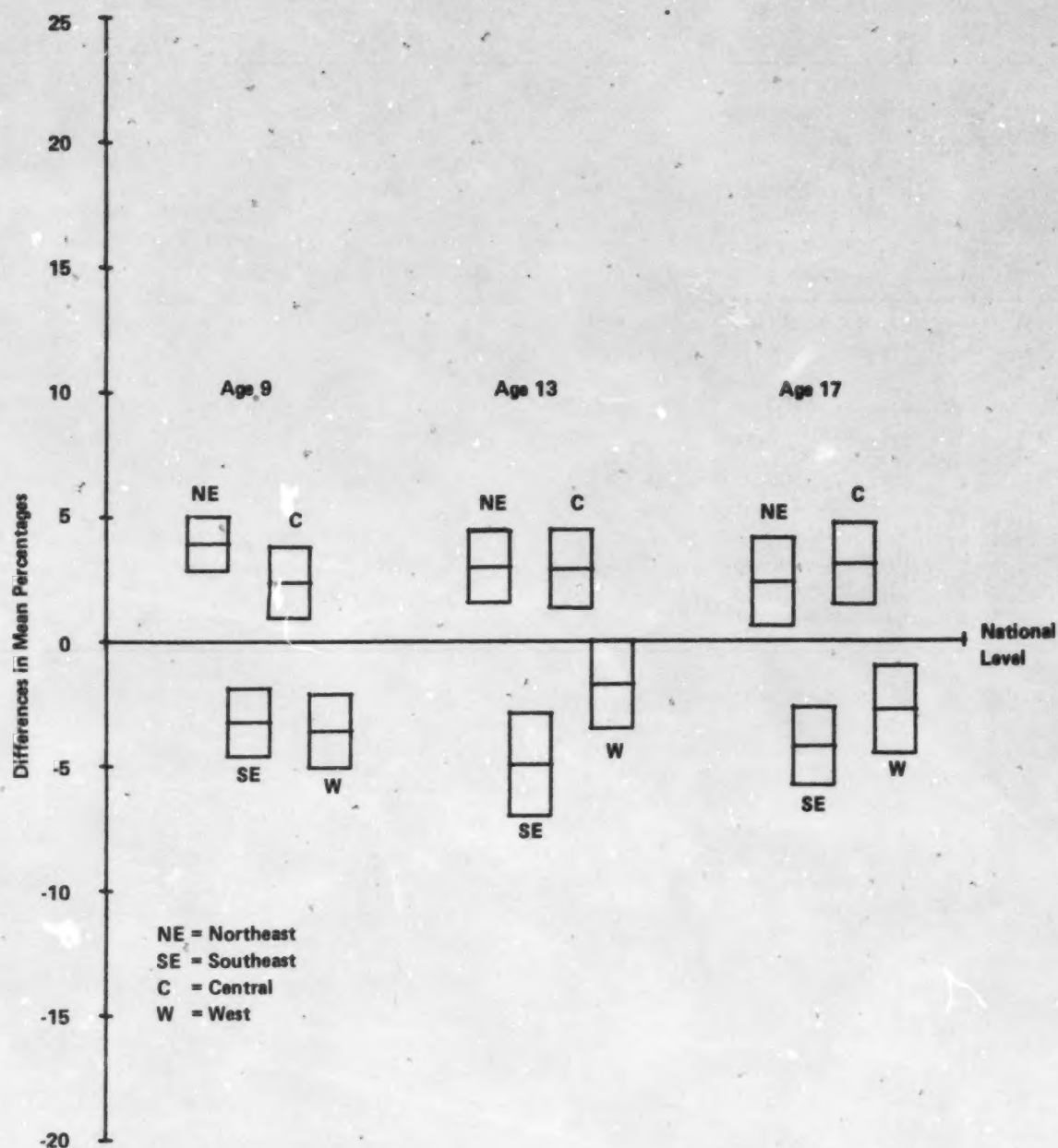


EXHIBIT 20. Differences in Mean Percentages Between the Sexes and the Nation on Understanding Items — Ages 9, 13 and 17

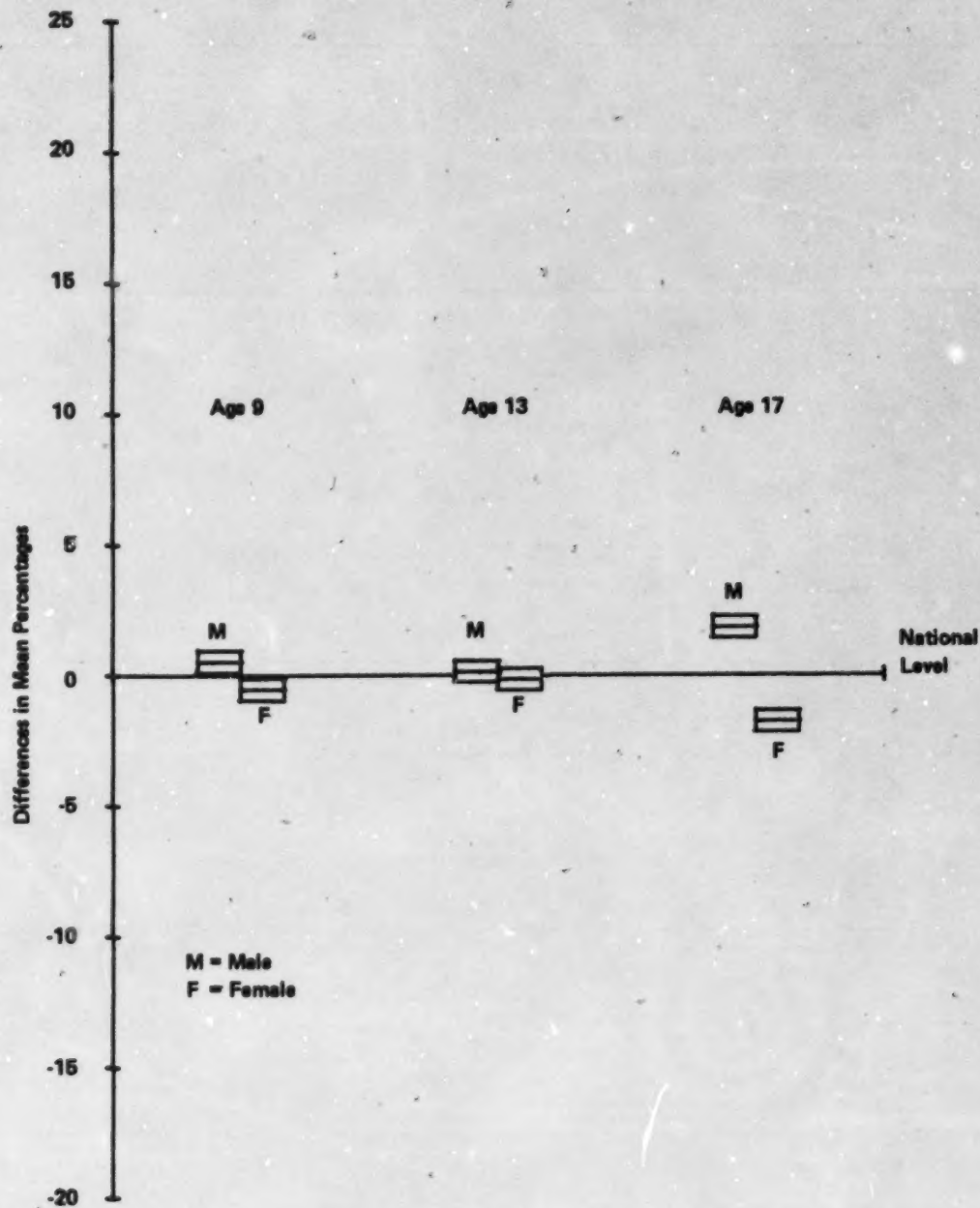


EXHIBIT 21. Differences in Mean Percentages Between Racial/Ethnic Groups and the Nation on Understanding Items – Ages 9, 13 and 17

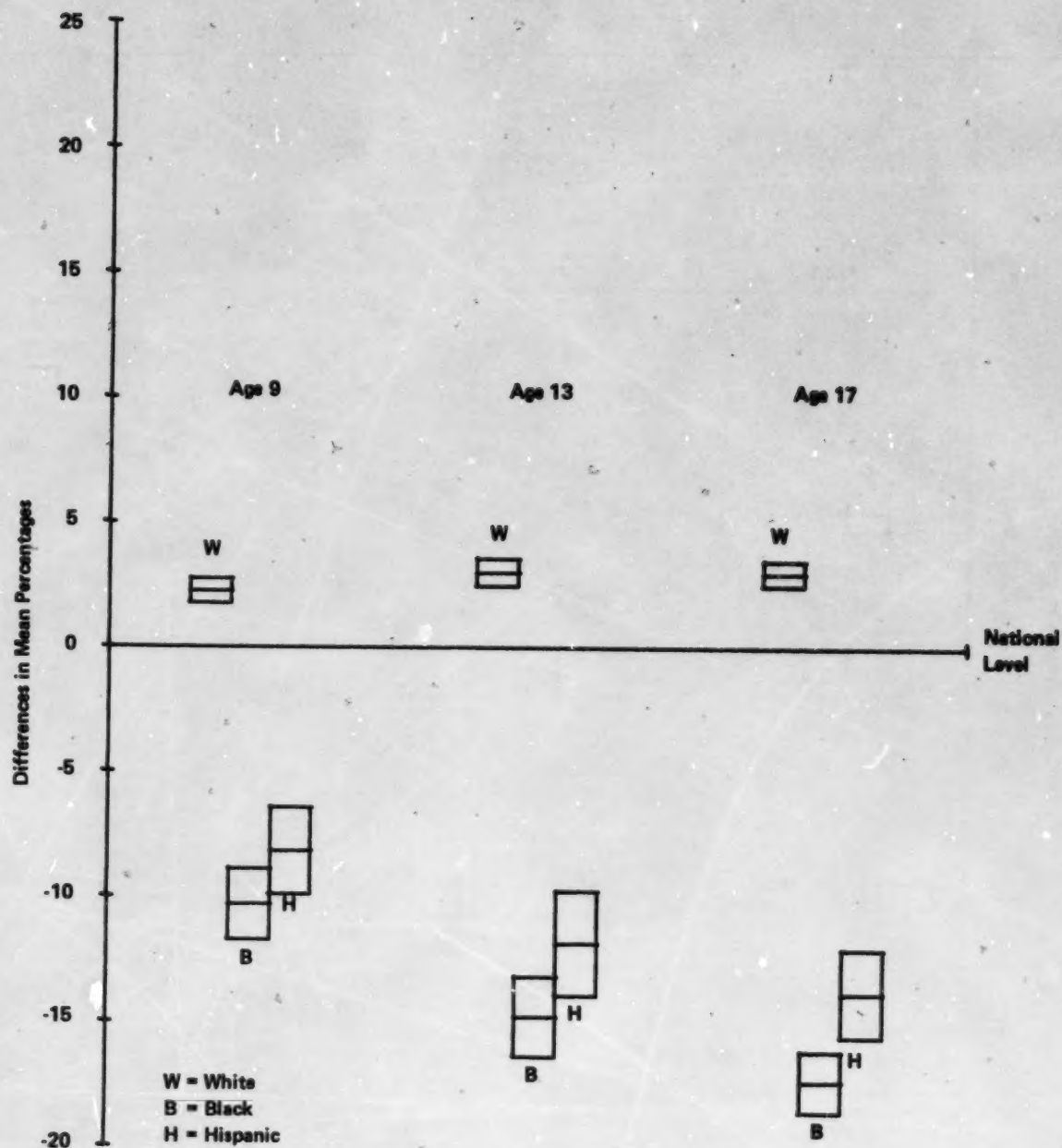


EXHIBIT 22. Differences in Mean Percentages Between Parental-Education Groups and the Nation on Understanding Items – Ages 9, 13 and 17

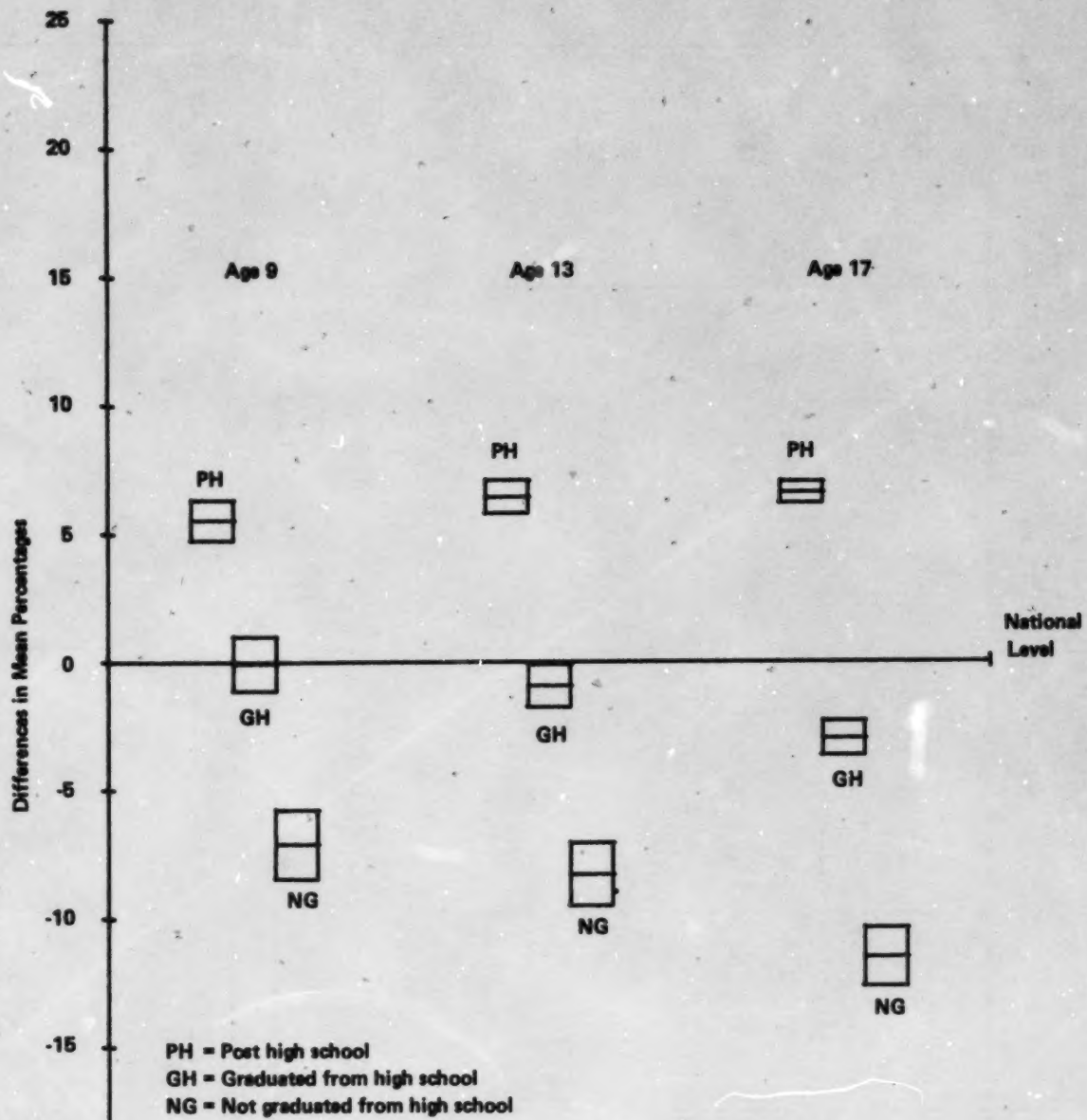


EXHIBIT 23. Differences in Mean Percentages Between Type-of-Community Groups and the Nation on Understanding Items — Ages 9, 13 and 17

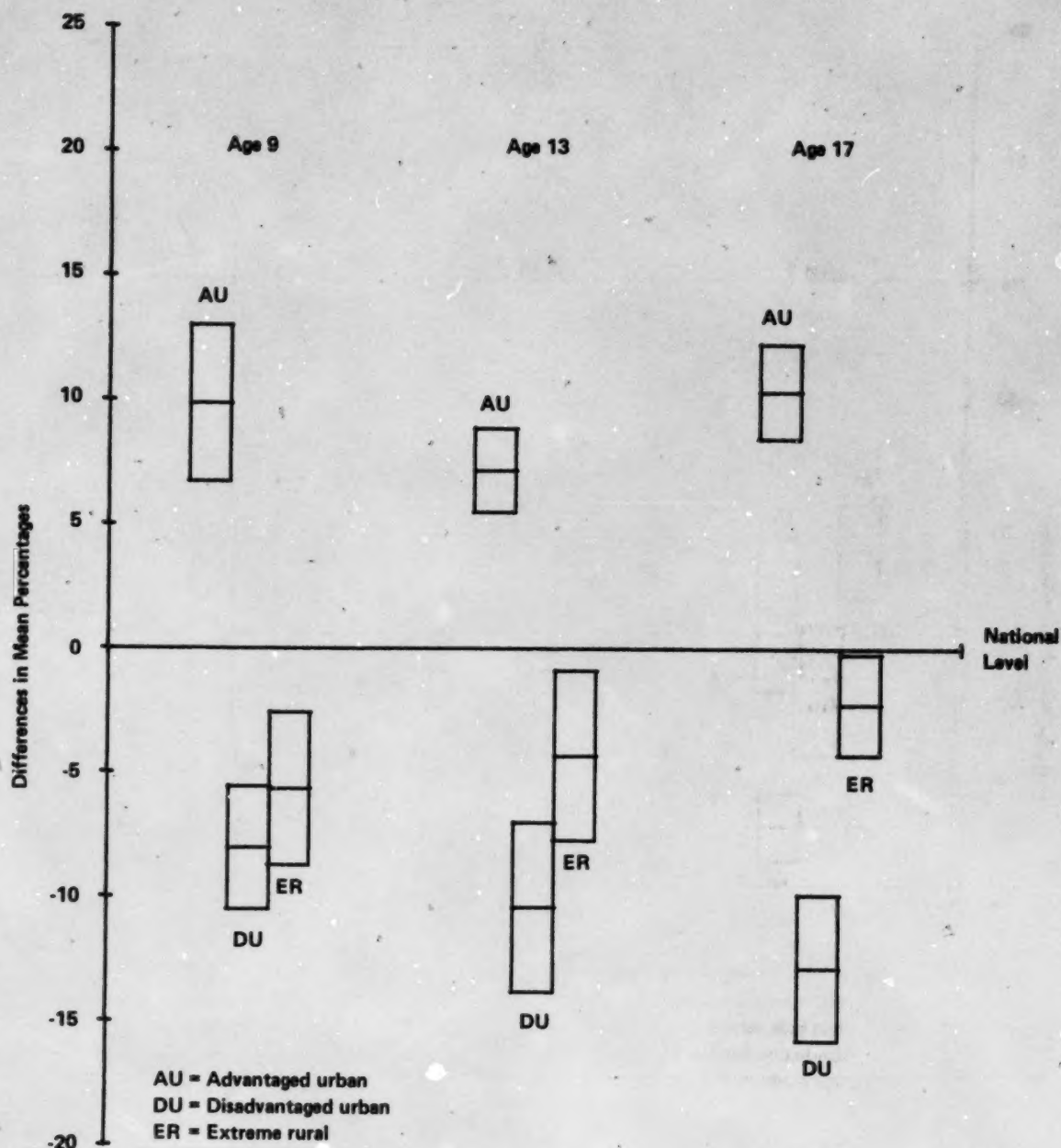


EXHIBIT 24. Differences in Mean Percentages Between Size-of-Community Groups and the Nation on Understanding Items – Ages 9, 13 and 17

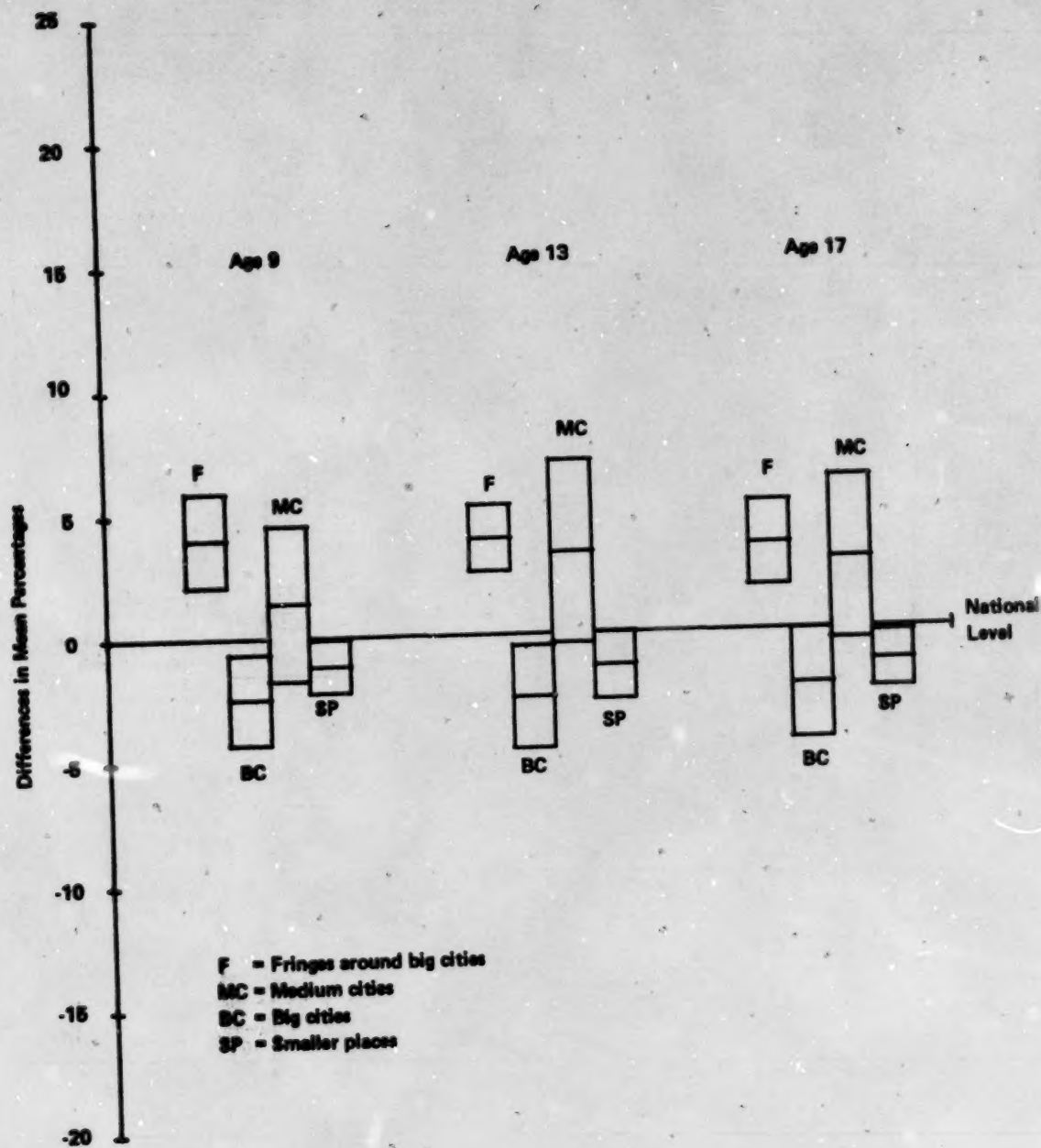
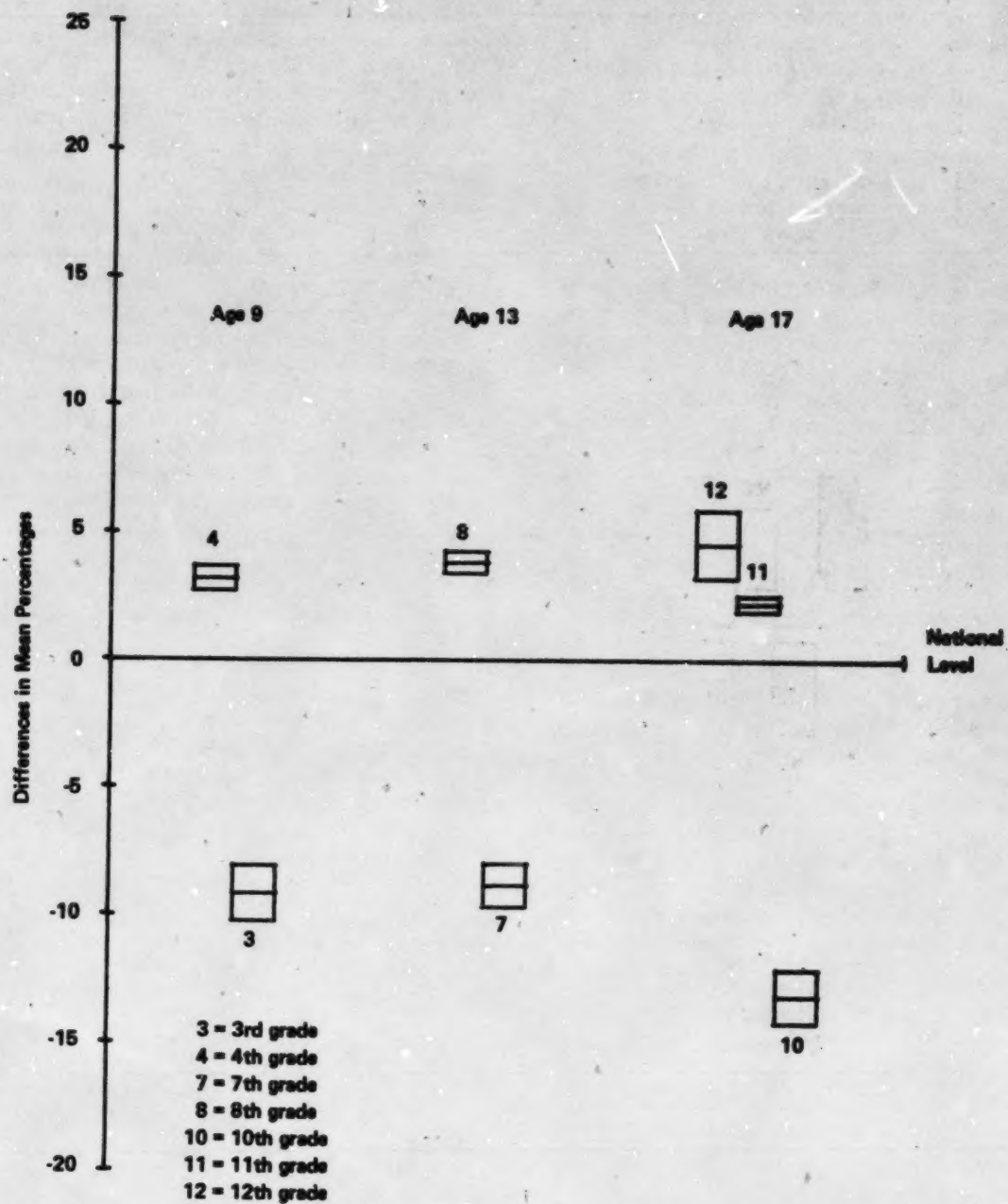


EXHIBIT 25. Differences in Mean Percentages Between Grade-in-School Groups and the Nation on Understanding Items – Ages 9, 13 and 17



(DU) and extreme rural (ER). As shown in Exhibit 23, the advantaged-urban groups performed consistently above the national level on understanding items, while the other two groups performed below the national level. There was a tendency for achievement of disadvantaged-urban students, relative to the nation, to decline with age, while that of extreme-rural students tended to improve. The performance difference between advantaged- and disadvantaged-urban students at age 17 was approximately 23 percentage points.

Size of community. Four kinds of communities defined solely on the basis of size are: fringes around big cities (F), big cities (BC), medium cities (MC) and smaller places (SP). The fringes around big cities were above the national level at each age, and big cities and small places were generally below the nation (Exhibit 24). Medium cities did not differ significantly from the nation, although means tended to be above national levels.

Schooling Variables

Grade in school. In the National Assessment sample, about three-fourths of the 9-year-olds were in the 4th grade (4) and one-fourth in the 3rd grade (3); about three-fourths of the 13-year-olds were in the 8th grade (8) and one-fourth in the 7th grade (7); and about 75% of the 17-year-olds were in the 11th grade (11), with about 15% in the 10th (10) and 10% in the 12th (12) grades. As one would expect, those in a higher grade at each age level showed higher mathematics achievement than those in a lower grade (Exhibit 25).

Mathematics courses taken (17-year-olds). Seventeen-year-olds were asked to supply information on mathematics courses they had taken. Exhibit 26 shows the percentages who reported that they had taken each of the various courses.

Mathematics achievement on understanding was analyzed for the following groups of

17-year-olds: (1) those who had not taken any of the courses listed in Exhibit 26 (<Alg I); (2) those who had taken Algebra I but no more advanced courses (Alg I); (3) those who had taken Geometry but not Algebra II or any more advanced courses (Geom); (4) those who had taken Algebra II but not Trigonometry, Probability and Statistics, or Pre-calculus (Alg II); (5) those who had taken Algebra II and Trigonometry, or Probability and Statistics, or Pre-calculus (>Alg II).

Exhibit 27 shows performance for the different levels of mathematics course-taking experience; approximately 36 percentage points separated the "extreme" groups. One must be careful, however, not to conclude that participation in more advanced mathematics is the sole cause of higher performance on assessment exercises. Other factors are involved — for example, many of the students who enroll in the higher-level mathematics courses are likely to be the more academically able students.

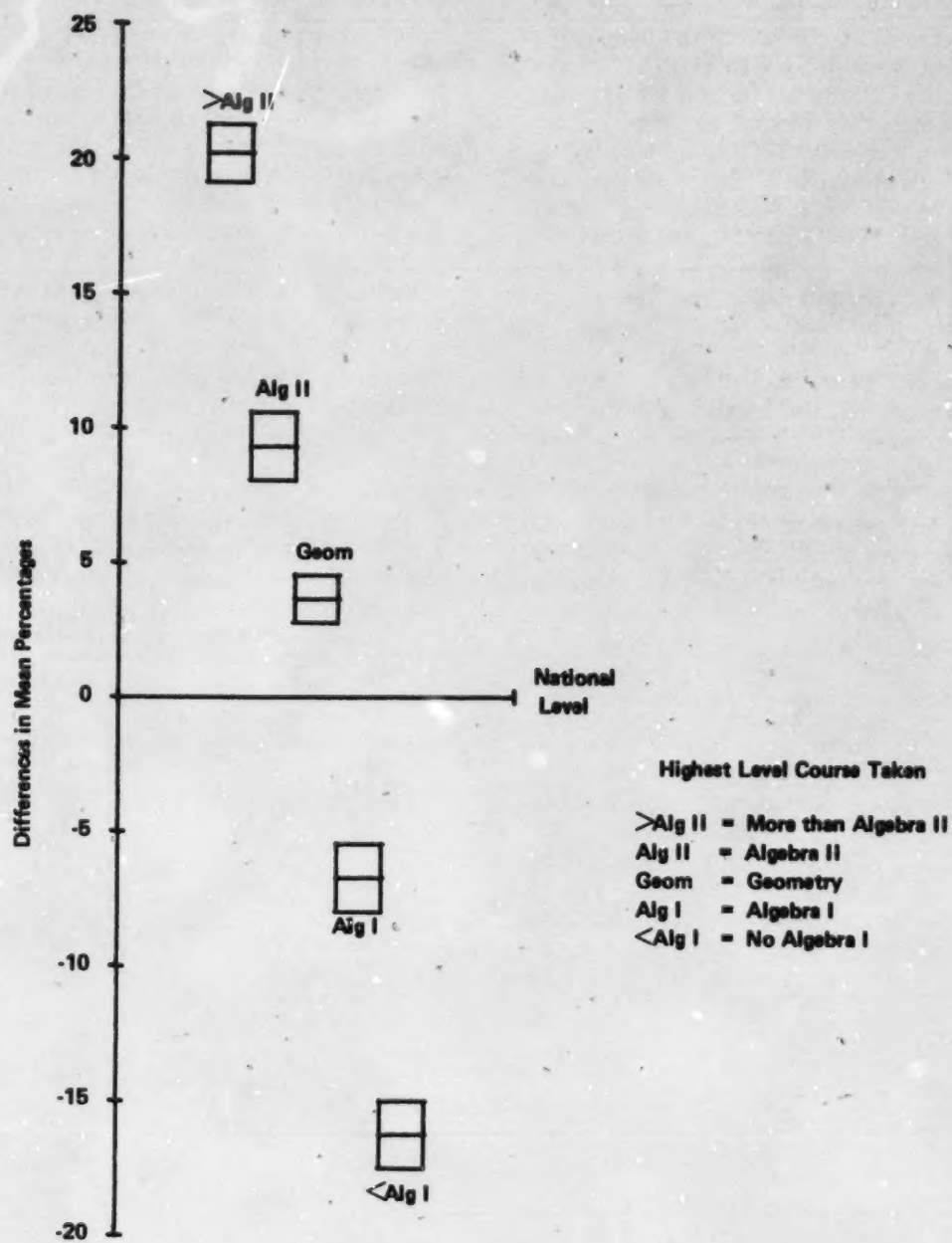
Age-Level Comparisons

The achievement levels of the three age groups assessed by National Assessment are compared in this section. Previous group comparisons were based on all understanding items administered to a particular age group; comparisons in this section are based on identical sets of items taken by two age groups. Nine- and 13-year-olds responded to 35 identical understanding items, while 13- and

EXHIBIT 26. Percentages of 17-Year-Olds Taking Various Mathematics Courses

	Percent Who Reported They Had Taken Course for ½ or 1 Year
Algebra I	72
Geometry	51
Algebra II	37
Trigonometry	13
Probability and statistics	3
Pre-calculus	4

EXHIBIT 27. Differences in Mean Percentages Between Groups Based on Mathematics-Course Background and the Nation on Understanding Items – Age 17



17-year-olds answered 80 understanding exercises that were the same.

As one might expect, performance improved between ages 9 and 13 (27 percentage points) and between ages 13 and 17 (13 percentage points). Exhibits 28 and 29 compare the performance of selected reporting groups for two age levels on identical understanding items. Thirteen-year-olds uniformly showed higher achievement in mathematical understanding than 9-year-olds (Exhibit 28); the same pattern did not hold true for 13- and 17-year-olds (Exhibit 29). The assessment found that, on the average:

- A higher percentage of white 13-year-olds answered understanding items correctly than did 17-year-old blacks and Hispanics.
- On understanding items, 13-year-olds whose parents had some post-high-school education outperformed 17-year-olds whose parents did not graduate from high school.
- Thirteen-year-olds in advantaged-urban

communities did better on understanding exercises than did 17-year-olds in disadvantaged-urban communities.

The improvement between 9- and 13-year-olds was very similar for most reporting groups analyzed. However, 13-year-old blacks, Hispanics and disadvantaged-urban students were farther below the national level than their 9-year-old counterparts; thus, improvement between ages 9 and 13 for these groups was less than the national improvement.

Similarly, most reporting groups showed about the same amount of improvement between ages 13 and 17. Again, however, improvements for blacks, Hispanics and disadvantaged-urban students between ages 13 and 17 were smaller than the national improvement.

These findings suggest that not only are the achievement levels of racial/ethnic minorities and disadvantaged students in this country below majority and advantaged groups, but that the deficit increases as students progress through school.

EXHIBIT 28. Average Percentages of Correct Response on Identical Sets of Understanding Items for Selected Reporting Groups – Ages 9 and 13

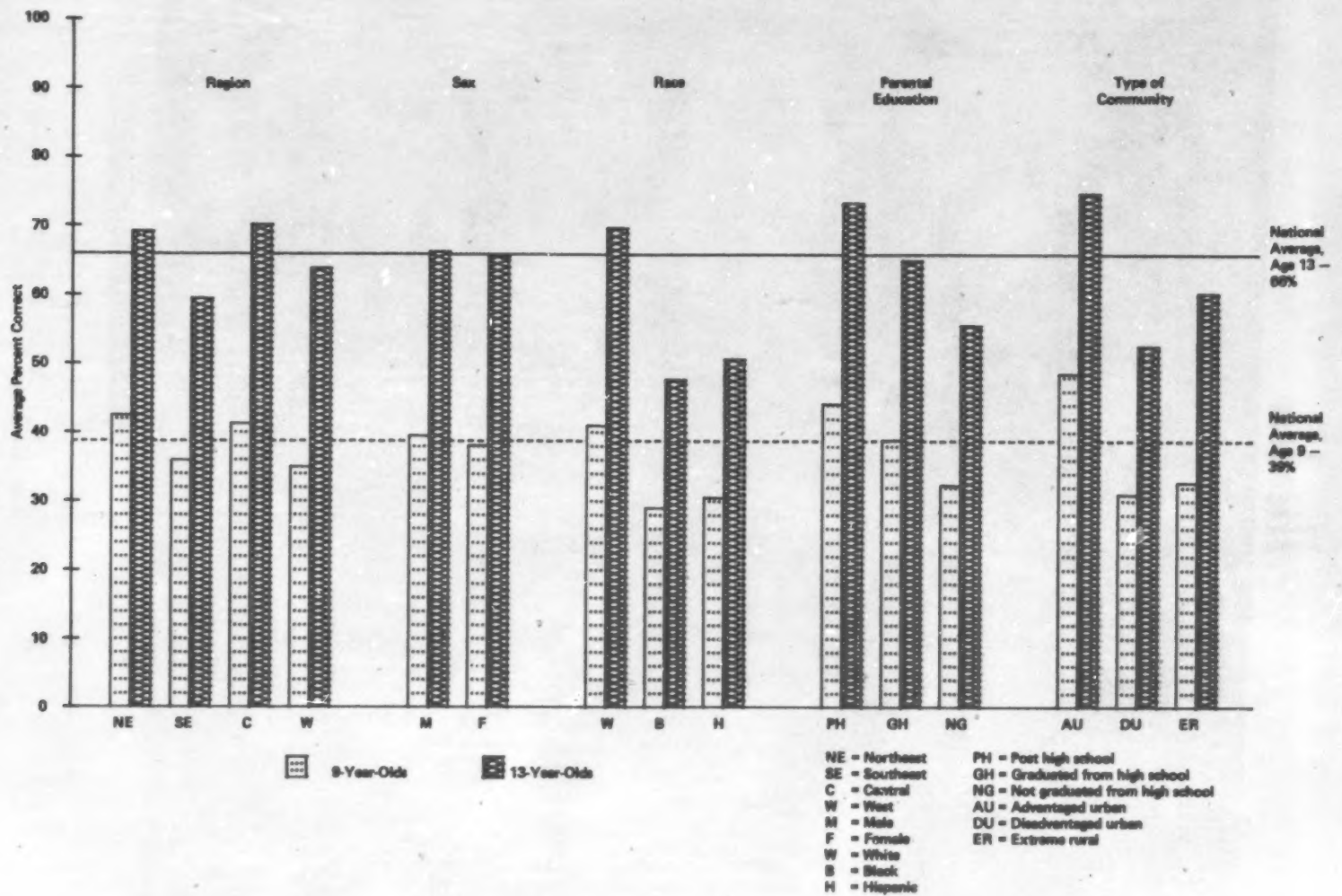
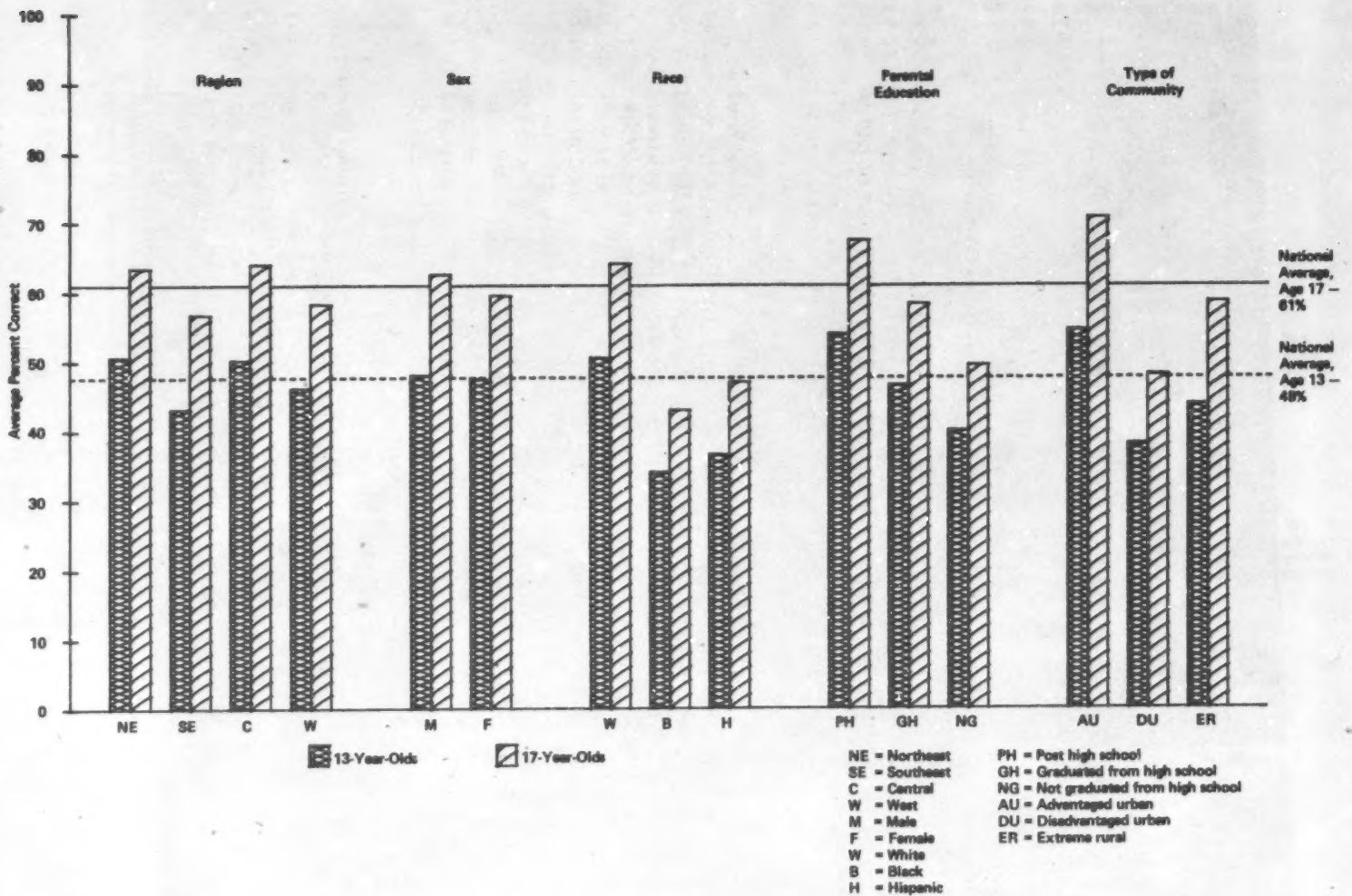


EXHIBIT 29. Average Percentages of Correct Response on Identical Sets of Understanding Items for Selected Reporting Groups – Ages 13 and 17



CHAPTER 3

OBSERVATIONS ON THE RESULTS

The 1977-78 national assessment of mathematics provides a comprehensive survey of students' present levels of mathematics achievement. The extensive data described in this report are best interpreted in light of current expectations and trends in mathematics education. Accordingly, National Assessment convened a panel of persons active in the field to interpret assessment results and to consider the implications for mathematics education.

Panel members included: Iris Carl, mathematics resource center specialist and curriculum writer, Houston (Texas) Independent School District, and member of the National Council of Teachers of Mathematics (NCTM) Program and Instructional Affairs Committees; Thomas Carpenter, associate professor of curriculum and instruction, University of Wisconsin, Madison; Robert Dilworth, professor of mathematics, California Institute of Technology, Pasadena; Edgar Edwards Jr., associate director of mathematics (K-12), Virginia Department of Education, Richmond, and member of the NCTM Board of Directors; and James W. Wilson, professor and department head, Mathematics Education Department, University of Georgia, Athens, and member of the NCTM Board of Directors.

This chapter presents selected quotes and a summary of comments made during a two and one-half day meeting at which the panel informally reviewed and discussed mathematics results. Observations pertain to data for all four cognitive process levels reported upon for the second assessment of mathematics and are not limited only to data for understanding items. Comments of the panel members represent their opinions as in-

dividuals and do not necessarily reflect the positions of any organizations with which they are associated.

Panel members emphasized cautions that must be kept in mind when interpreting the data. The format of and the vocabulary used in an item can have an influence on performance. For example, multiple-choice items are often easier to answer than open-ended items. Specific vocabulary may present difficulties, as in the case in which many more students could calculate the "distance around" a figure than could find the "perimeter" of the same figure. Thus, in generalizing from performance on an item to ability to perform a task, one must take into account the nature of the particular items used in the assessment. In addition, although students are told that the assessment is not a test and that their scores will not be recorded, the assessment is a timed, test-like situation, and responses given under the stress of the situation may not be totally accurate indicators of ability.

A Historical Perspective

Amid reports of declining test scores and public disenchantment with curricular and teaching innovations, there has been widespread demand during the 1970s that education go "back to the basics." A number of states and local districts have instituted "minimal-competency" requirements for high school graduates, and schools increasingly have been held "accountable" for the performance of their students.

In mathematics, the movement to "basics" has often resulted in a narrowing of the cur-

riculum, stressing computational skills and knowledge of facts and definitions at the expense of higher-order abilities. Wilson noted that textbooks have changed during the 1970s: "By the mid-1970s, textbooks in the schools emphasized computational skills. Word problems were streamlined to present students with the least possible amount of complexity."

The increased testing associated with minimal competency and accountability movements has had an effect as well. Noting that knowledge and computational skills are relatively easy to teach and to test, Edwards said, "There is a danger that increased concern over test scores may lead to concentration on training students to answer questions on tests at the expense of teaching other important mathematical concepts and skills."

Training of mathematics teachers is not consistent across the country, and some persons teaching mathematics may not have received sufficient training. For example, elementary teachers may not be required to take any mathematics beyond what they have had in high school. Dilworth explained that this can lead to a fear of teaching mathematics, and described a group of elementary teachers in California who "felt very insecure in teaching mathematics." Edwards stated that many colleges do not offer methods courses specifically designed for prospective teachers of mathematics. He added that "although teacher preparation is not necessarily directly related to student performance, too often, new teachers are simply given a textbook and little or no guidance on effective teaching methods."

Implications of the Results

The panel members were generally satisfied with performance on items assessing knowledge. Wilson summarized the group's feeling about knowledge achievement when he said, "Knowledge levels reflect the emphasis placed on topics in the mathematics curriculum, and performance is generally very satisfactory." In regard to computational skills, the panel felt

that achievement on whole number computation items was adequate. Carpenter stated, "Performance with whole numbers is pretty good; computation with other types of numbers falls off."

The panel felt that the National Assessment results provide substantial evidence that higher-level cognitive abilities are not being adequately developed. In recent years, curriculum and instruction have encouraged students to use "mechanical" approaches to perform mathematical tasks, whether the tasks are merely the demonstration of skills or the solving of problems. Consequently, the understanding of mathematical concepts that would facilitate the performance of many mathematical tasks appears to be lacking. Carpenter noted that "often, when students had difficulty with skills, it seemed to be because they did not understand the concepts involved."

Results on exercises involving fractions illustrate students' reliance on mechanical processes as well as their lack of understanding of certain concepts. On an item requiring students to add two simple common fractions, $\frac{3}{4}$ plus $\frac{1}{2}$, many students (34% of the 13-year-olds and 15% of the 17-year-olds) added numerator to numerator and denominator to denominator. The panel found it particularly striking that only 24% of the 13-year-olds and 37% of the 17-year-olds realized that $\frac{12}{13} + \frac{7}{8}$ equals approximately 2. Dilworth stated that many students apparently do not understand the concept of fractions and mechanically tried to add the numbers in the limited time available, thus selecting unreasonable answers such as 19 and 21. Carl noted that "although students may have skill in using distinct algorithms, the depth of understanding of the concepts required to effectively apply skills in various situations appears to be missing." She also observed that, when asked to estimate, many students tried to calculate answers exactly and then rounded them off, thus defeating the purpose of estimation.

Computation exercises involving decimal fractions show that when students are faced with routine addition or subtraction items in

which the numerals involved are already "lined up," most students are successful. However, when the numerals are not already aligned for computation, many students show either a lack of understanding of, or a disregard for, place value.

Noting that a number of the items dealing with percentage were not difficult if one had an understanding of the concept of percent (for example, "30 is what percent of 60?"), the group felt that more emphasis must be placed on understanding and working with percentage in a problem-solving context. The panel felt that students often learn the algorithms for each of the "cases" associated with percent and that students memorize (and subsequently forget) the situations in which each formula is appropriate. Wilson said, "The primary emphasis in teaching percent in recent years, focusing on mechanical application of computational skills, has not led to adequate performance."

Similarly, exercises on which students could use various properties of operations to their advantage in order to arrive at a solution quickly showed that students either do not understand the properties or they just tend not to use them. For example, $(37 \times 5) \times 2$ is the same as $37 \times (5 \times 2)$, yet only about one-third of the 13-year-olds and about one-half of the 17-year-olds multiplied correctly in the time allowed when shown the problem in the first form.

Almost all students at all three age levels were able to recognize a rectangle. However, on exercises concerning the properties of figures, many students showed a lack of understanding of certain geometric concepts. On one exercise, approximately half of the 13- and 17-year-olds indicated that opposite sides of equal length is a sufficient condition for a figure to be a rectangle. The results on another exercise showed that approximately two-fifths of the 13-year-olds and one-third of the 17-year-olds felt that parallel opposite sides is a sufficient condition for a figure to be a rectangle.

Performance in making measurements gen-

erally was regarded as satisfactory. However, many exercises involving measurement, like exercises dealing with other topics, reveal that students are not very flexible — they tend not to transfer their knowledge and skills to unusual or unfamiliar situations. For example, although National Assessment data show that at least 60% at each age level understood that the sides of a square are equal in length and about one-half of the 13-year-olds and 70% of the 17-year-olds calculated the area of a rectangle correctly when given the length and width, only 12% of the 13-year-olds and 42% of the 17-year-olds figured out how to find the area of a square when the dimension of only one side was given. Carpenter said, "Youngsters don't think about problems. They search for ways to apply the mechanical approaches that they have been taught, and if anything out of the ordinary appears in the problem, they are at a loss."

The consultants felt that the narrowing of the curriculum in response to the back-to-the-basics movement has resulted in insufficient attention to higher-level processes, including understanding and problem solving. In emphasizing lower-level knowledge and skills, the present curriculum tends to lose sight of the fact that skills should not be solely ends in themselves, but should also be regarded as tools to be applied in problem-solving situations. Furthermore, ability with higher-level processes such as understanding and "thinking through" problems can often assist one in performing skill-level tasks.

Often students are not allowed to go on to higher-level processes until they have completely mastered lower-level ones. The panel felt that this could severely limit students' exposure to a wide range of mathematical topics and could result in students having very little idea of how the various things they had learned were related. Carl noted that in the Houston schools, programs that require students to move at a set pace often produce better results than those that insist on complete mastery of one topic before moving on to another.

In examining results for population groups,

panel members expressed some surprise that differences between various groups were similar for each of the four cognitive process levels assessed (knowledge, skill, understanding and application). The patterns of relative performance of males and females did vary somewhat across age and cognitive process levels. However, performance differences between the sexes were small when compared with differences between racial groups and groups based on types of community.

The interpretive panel acknowledged that many factors have an influence on discrepancies in group performance and felt that not enough information was available to single out particular ones. They did feel, however, that the data present strong evidence that inequities in American education still remain.

Recommendations

In light of the data reported by National Assessment, the panel felt that a number of recommendations are in order:

1. An expanded definition of what is "basic" in mathematics is crucial to foster students' ability to cope with different types of mathematical tasks. Students must be involved in activities

requiring higher-level, as well as lower-level, cognitive processes.

2. There is a need to promote understanding of concepts and to link skill development to problem-solving activities. Mechanical, algorithmic approaches to higher-level tasks should not be encouraged.
3. Success on tests stressing low-level cognitive processes should not be the only criterion used to measure the effectiveness of mathematics programs. Too often the tests then become the objective, and important areas that require more effort and time to teach fall by the wayside.
4. There is a need for more consistent and comprehensive teacher education in the mathematics area. More attention should be devoted to preparing teachers to teach concepts and problem solving so they will not tend to rely as heavily on drill and memorization work.
5. There is a need for further research into causes and potential remedies for the disparities in group performance that presently exist.

APPENDIX A

DEFINITIONS OF REPORTING GROUPS

National Assessment examines results for various groups of people within the national population. These groups are defined as follows.

Background Variables

Region

The country has been divided into four regions: Northeast, Southeast, Central and West. States included in each region are shown on the following map.



Sex

Results are reported for males and females.

Race

Results are presented for blacks, whites and Hispanics.

Level of Parental Education

Three categories of parental-education levels are defined by National Assessment, based on students' reports. These categories are: (1) those whose parents did not graduate from high school, (2) those who have at least one parent who graduated from high school and (3) those who have at least one parent who has had some post-high-school education.

Type of Community

Communities in this category are defined by an occupational profile of the area served by a school, as well as by the size of the community in which the school is located.

Advantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 200,000 where a high proportion of the residents are in professional or managerial positions.

Disadvantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 200,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme-rural communities. Students in this group attend schools in areas having a population under 10,000 where most of the residents are farmers or farm workers.

Size of Community

Big cities. Students in this group attend schools within the city limits of cities having a 1970 census population over 200,000.

Fringes around big cities. Students in this group attend schools within metropolitan areas (1970 U.S. Bureau of the Census urbanized areas) served by cities having a population greater than 200,000, but outside the city limits.

Medium cities. Students in this group attend schools in cities having a population between 25,000 and 200,000, not classified in the fringes-around-big-cities category.

Smaller places. Students in this group attend schools in communities having a population less than 25,000, also not classified in the fringes-around-big-cities category.

Schooling Variables

Grade Level

Results are categorized for 9-year-olds in

the 3rd or 4th grade; 13-year-olds in the 7th or 8th grade; and 17-year-olds in the 10th, 11th or 12th grade.

Mathematics Courses Taken by 17-Year-Olds

Seventeen-year-olds were asked about their mathematics course-taking experiences. Five levels of mathematics-course experience were defined: (1) those who had not taken Algebra I or any more advanced mathematics courses; (2) those who had taken Algebra I but no more advanced courses; (3) those who had taken Geometry but not Algebra II or any more advanced courses; (4) those who had taken Algebra II but no more advanced courses (Trigonometry, Probability and Statistics, or Pre-calculus); (5) those who had taken Algebra II and at least one more advanced course — Trigonometry, Probability and Statistics, or Pre-calculus. A student was said to have taken a course only if he or she had taken it for one-half year or more.

APPENDIX B

GROUP DIFFERENCES FROM THE NATION ON MATHEMATICAL UNDERSTANDING ITEMS

TABLE B-1. Average Group Differences From the Nation for
Understanding Items — Ages 9, 13 and 17

	Age 9	Age 13	Age 17
Number of items	44	108	105
National percentage	39.6	51.7	58.0
Region			
Northeast	4.0*	3.0*	2.4*
Southeast	-3.2*	-4.9*	-4.7*
Central	2.4*	2.9*	3.1*
West	-3.6*	-1.7	-2.8*
Sex			
Male	0.5*	0.1	1.8*
Female	-0.5*	-0.2	-1.8*
Race			
White	2.3*	3.0*	3.0*
Black	-10.3*	-14.8*	-17.4*
Hispanic	-8.2*	-11.8*	-13.8*
Parental education			
Post high school	5.6*	6.5*	6.6*
Graduated high school	-0.1	-0.9*	-3.0*
Not graduated high school	-7.1*	-8.3*	-11.5*
Type of community			
Advantaged urban	9.9*	7.2*	10.4*
Disadvantaged urban	-8.0*	-10.4*	-12.8*
Extreme rural	-5.7*	-4.3*	-2.3*
Size of community			
Fringes around big cities	4.0*	3.9*	3.5*
Big cities	-2.5*	-2.5*	-2.2*
Medium cities	1.4	3.3	2.8
Smaller places	-1.2*	-1.3	-1.3*

TABLE B-1 (continued). Average Group Differences From the Nation for Understanding Items – Ages 9, 13 and 17

	Age 9	Age 13	Age 17
Grade			
3	-9.2*		
4	3.2*		
7		-8.8*	
8		3.9*	
10			-13.1*
11			2.2*
12			4.6*
Courses taken			
<Algebra I			-16.2*
Algebra I			-6.7*
Geometry			3.7*
Algebra II			9.3*
>Algebra II			20.2*

*Indicates mean percentages significantly different from the nation at the .05 level.

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